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Mathematics—Class 10

Chapter–1

Real Numbers

Important Points

1. **Euclid's Division Algorithm—**Any positive integer a can be divided by another positive integer b in such a way that it leaves a remainder r that is smaller than b.
2. **Euclid's Division Lemma—**Given two positive integers a and b we can find out the whole numbers q and r satisfying $a = bq + r$, $0 \leq r < b$, i.e., such numbers exist.
   Euclid was the first Greek mathematician who put forth a new ideology for the study of plane Geometry. According to him on dividing a positive integer by some other positive integer, we obtain the quotient q and the remainder r and the remainder r is either zero or smaller than the divisor b i.e, $0 \leq r < b$.
   In simple words
   Dividend (a) = Divisor (b) × Quotient (q) + Remainder (r)
3. Euclid's Division Algorithm is based on Euclid's Division Lemma. Using this technique we can compute the Highest Common Factor (HCF) of two given positive integers a and b ($a > b$). Following the steps given below:
   Step I : Apply Euclid's Division Lemma to find q and r where $a = bq + r$, $0 \leq r < b$.
   Step II : If $r = 0$, then HCF = b. If $r \neq 0$, then we apply Euclid's Division Lemma on b and r.
   Step III : Continue this process till the remainder is zero. The division at this stage is HCF (a, b). Also,
   HCF (a, b) = HCF (b, r)
   With the help of this main relation, other relations may also be written as given below :

   (i) $\text{H.C.F.} = \dfrac{a \times b}{\text{L.C.M.}}$

   (ii) $\text{L.C.M.} = \dfrac{a \times b}{\text{H.C.F.}}$

   (iii) $a = \dfrac{\text{H.C.F.} \times \text{L.C.M.}}{b}$

   (iv) $b = \dfrac{\text{H.C.F.} \times \text{L.C.M.}}{a}$
4. **Fundamental Theorem of Arithmetic—**Every composite number can be expressed (factorised) as a product of prime numbers and this factorisation is unique.
   For any two positive integers a and b, H.C.F. (a, b) × L.C.M. (a, b) = $a \times b$.
5. **Important Points—**

(i) Euclid's Division Algorithm is not only useful in computing the H.C.F. of large numbers but also important for the reason that this is one of more algorithms that were first of all used as a program in a computer.

(ii) Euclid's Division Lemma and Euclid's Division Algorithm are so closely interlinked that people often call Euclid's Division Lemma as Euclid's Division Algorithm.

(iii) Euclid's Division Lemma/Algorithm is stated only for positive integers. However it can be applied for all integers (except zero, i.e., $b \neq 0$).

6. If p is a prime number and if p divides a^2, then p will divide a, where a is a positive integer.

7. $\sqrt{2}, \sqrt{3}, \sqrt{5}$ and in general, $\sqrt{p}$ are irrational numbers, where p is a prime number.

8. Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form $\frac{p}{q}$, where p and q are co prime, and the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers.

9. Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then, x has a decimal expansion which terminates.

10. Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form $2^n 5^m$, where n, m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating (recurring).

Text Book Questions

EXERCISE 1.1

1. Use Euclid's division algorithm to find the HCF of :

(i) 135 and 225 (ii) 196 and 38220

(iii) 867 and 255

Solution : (i) 135 and 225

Using Euclid's Division Algorithm

Step I : $\because$ 225 > 135

Therefore according to Euclid's Division Lemma

$225 = 135 \times 1 + 90$

Step II : $\because$ Remainder $90 \neq 0$, therefore now applying Euclid's Division Lemma on 135 and 90

$135 = 90 \times 1 + 45$

Step III : $\because$ Remainder $45 \neq 0$, therefore now applying Euclid's Division Lemma on 90 and 45

$90 = 45 \times 2 + 0$

The remainder has now become zero, so this procedure stops. The divisor in this step is 45. Therefore, the HCF of 135 and 225 is 45. **Ans.**

(ii) 196 and 38220

Using Euclid's Division Algorithm

Step I : $\because$ 38220 > 196

Therefore according to Euclid's Division Lemma

$38220 = 196 \times 195 + 0$

Since the remainder obtained is zero. Therefore this procedure stops here. The divisor in this step is 196. Therefore, the HCF of 38220 and 196 is 196. **Ans.**

(iii) 867 and 255

Using Euclid's Division Algorithm

Step I : $\because$ 867 > 255

Therefore according to Euclid's Division Lemma

$867 = 255 \times 3 + 102$

Step II : $\because$ Remainder $102 \neq 0$, therefore now applying Euclid's Division Lemma on 255 and 102

$225 = 102 \times 2 + 51$

Step III : $\because$ Remainder $51 \neq 0$, therefore now applying Euclid's Division Lemma on 102 and 51

$102 = 51 \times 2 + 0$

The remainder has now become zero, so

this procedure stops. The divisor in this step III is 51. Therefore, the HCF of 867 and 255 is 51. **Ans.**

2. Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

Solution : Let a be a positive odd integer. Now for a and $b = 6$, by the application of Euclid's Division Algorithm $0 \le r < 6$, i.e., the values of a can be $6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$, where q is some quotient. Now since a is odd positive integer. Therefore it cannot be of the form $6q$, $6q + 2$ or $6q + 4$ since all these being divisible by 2 are even positive integers. Therefore, any positive odd integer is of the form $6q + 1$ or $6q + 3$ or $6q + 5$ where q is some integer.

3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution : Group of members in parade = 616 and 32

According to the question, we are to find out the maximum number of columns, i.e., we have to find out the HCF of 616 and 32.

Step I : $\because$ 616 > 32

Therefore according to Euclid's Division Lemma

$616 = 32 \times 19 + 8$

Step II : $\because$ Remainder $8 \neq 0$, therefore now applying Euclid's Division Lemma on 32 and 8

$32 = 8 \times 4 + 0$

Since the remainder obtained now is zero. Therefore this procedure stops. The divisor in this step is 8. Therefore, the HCF of 616 and 32 is 8, i.e., the maximum number of columns in which they can march in parade is 8. **Ans.**

4. Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.

[**Hint :** Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]

Solution : Let x be any positive integer. Then it can be of form $3q$, $3q + 1$ or $3q + 2$.

Therefore if $x = 3q$

squaring both sides

$$x^2 = (3q)^2 = 9q^2 = 3(3q^2) = 3m$$

where $m = 3q^2$ and m is also an integer.

Hence $x^2 = 3m$ ....(i)

Again if $x = 3q + 1$

squaring both sides

$x^2 = (3q + 1)^2$

$\Rightarrow$ $x^2 = 9q^2 + 2 \times 3q \times 1 + 1$

$\Rightarrow$ $x^2 = 3(3q^2 + 2q) + 1$

$\Rightarrow$ $x^2 = 3m + 1$ .....(ii)

Where $m = 3q^2 + 2q$ and m is also an integer.

In the last if $x = 3q + 2$

squaring both sides

$$x^2 = (3q + 2)^2 = 9q^2 + 12q + 4 = 3(3q^2 + 4q + 1) + 1 = 3m + 1 \quad .....(iii)$$

Where $m = 3q^2 + 4q + 1$ and m is also an integer.

Hence from (i), (ii) and (iii)

$x^2 = 3m$ or $3m + 1$

Therefore the square of any positive integer is of the form $3m$ or $3m + 1$ for some integer m. **Ans.**

5. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

Solution : Let a be any positive integer and $b = 3$.

$\therefore \quad a = 3q + r$

Where q is the quotient and r is the remainder.

Here $\quad 0 \le r < 3$

Therefore if $r = 0$ then $a = 3q$

if $r = 1$ then $a = 3q + 1$

if $r = 2$ then $a = 3q + 2$

i.e., a is of the form $3q$, $3q + 1$ or $3q + 2$.

Now if $a = 3q$

Cubing both sides

$$a^3 = (3q)^3$$

$$a^3 = 27q^3 = 9\,(3q^3) = 9m$$

Here $\quad m = 3q^3$ and m is also an integer

$\therefore \quad a^3 = 9m \quad$ .....(i)

Again if $\quad a = 3q + 1$

Cubing both sides

$$a^3 = (3q + 1)^3$$

$$\Rightarrow \quad a^3 = 27q^3 + 27q^2 + 9q + 1$$

$$= 9\,(3q^3 + 3q^2 + q) + 1$$

$$= 9m + 1$$

Here $\quad m = 3q^3 + 3q^2 + q$ and m is also an integer.

Hence $\quad a^3 = 9m + 1 \quad$ .....(ii)

Now if $\quad a = 3q + 2$

Cubing both sides

$$a^3 = (3q + 2)^3$$

$$= 27q^3 + 54q^2 + 36q + 8$$

$$a^3 = 9\,(3q^3 + 6q^2 + 4q) + 8$$

$$a^3 = 9m + 8 \quad .... \text{(iii)}$$

Here $\quad m = 3q^3 + 6q^2 + 4q$ and m is also an integer.

Hence $\quad a^3 = 9m + 8$

Now from (i), (ii) and (iii) we find that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

EXERCISE 1.2

1. Express each number as a product of its prime factors :

(i) 140 (ii) 156 (iii) 3825

(iv) 5005 (v) 7429

Solution—

(i) Prime factors of 140 $= 2 \times 70$

$= 2 \times 2 \times 35$

$= 2 \times 2 \times 5 \times 7$

$= 2^2 \times 5 \times 7$

(ii) Prime factors of 156 $= 2 \times 78$

$= 2 \times 2 \times 39$

$= 2 \times 2 \times 3 \times 13$

$= 2^2 \times 3 \times 13$

(iii) Prime factors of 3825 $= 3 \times 1275$

$= 3 \times 3 \times 425$

$= 3 \times 3 \times 5 \times 85$

$= 3 \times 3 \times 5 \times 5 \times 17$

$= 3^2 \times 5^2 \times 17$

(iv) Prime factors of 5005 $= 5 \times 1001$

$= 5 \times 7 \times 143$

$= 5 \times 7 \times 11 \times 13$

(v) Prime factors of 7429 $= 17 \times 437$

$= 17 \times 19 \times 23$

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91 (ii) 510 and 92

(iii) 336 and 54

Solution : (i) 26 and 91

Prime factors of 26 $= 2 \times 13$

Prime factors of 91 $= 7 \times 13$

$\therefore$ LCM of 26 and 91 $= 2 \times 7 \times 13 = 182$

and HCF of 26 and 91 = 13

Verification—HCF (26, 91) × LCM (26, 91)

$= 13 \times 182$

$= 13 \times 2 \times 91$

$= 26 \times 91$

= Product of given numbers

(ii) 510 and 92

Prime factors of 510 $= 2 \times 255$

$= 2 \times 3 \times 85$

$= 2 \times 3 \times 5 \times 17 \quad$ .....(i)

and Prime factors of 92 $= 2 \times 46$

$= 2 \times 2 \times 23$

$= 2^2 \times 23 \quad$ .....(ii)

LCM (510, 92) $= 2^2 \times 3 \times 5 \times 17 \times 23$
$= 23460$
and HCF (510, 92) = 2

Verification—

HCF (510, 92) × LCM (510, 92)
$= 2 \times 23460$
$= 2 \times 2^2 \times 3 \times 5 \times 17 \times 23$
$= 2 \times 3 \times 5 \times 17 \times 2^2 \times 23$
$= 510 \times 92$
= Product of given numnbers

(iii) 336 and 54

Prime factors of 336 $= 2 \times 168$
$= 2 \times 2 \times 84$
$= 2 \times 2 \times 2 \times 42$
$= 2 \times 2 \times 2 \times 2 \times 21$
$= 2 \times 2 \times 2 \times 2 \times 3 \times 7$
$= 2^4 \times 3 \times 7$

Prime factors of 54 $= 2 \times 27$
$= 2 \times 3 \times 9$
$= 2 \times 3 \times 3 \times 3$
$= 2 \times 3^3$

$\therefore$ HCF (336, 54) $= 2 \times 3 = 6$
LCM $= 2^4 \times 3^3 \times 7$
$= 3024$

Verification—

HCF (336, 54) × LCM (336, 54)
$= 6 \times 3024$
$= 2 \times 3 \times 2^4 \times 3^3 \times 7$
$= 2^4 \times 3 \times 7 \times 2 \times 3^3$
$= 336 \times 54$
= Product of given numbers

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29
(iii) 8, 9 and 25

Solution : (i) 12, 15 and 21

Prime factors of 12 $= 2 \times 2 \times 3$
Prime factors of 15 $= 3 \times 5$
Prime factors of 21 $= 3 \times 7$
$\therefore$ LCM (12, 15 and 21) $= 2^2 \times 3 \times 5 \times 7$
$= 420$ **Ans.**
and HCF (12, 15 and 21) = 3 **Ans.**

(ii) 17, 23 and 29

Prime factors of 17 $= 1 \times 17$
Prime factors of 23 $= 1 \times 23$
Prime factors of 29 $= 1 \times 29$
$\therefore$ LCM (17, 23 and 29) $= 17 \times 23 \times 29$
$= 11339$ **Ans.**
and HCF (17, 23 and 29) = 1 **Ans.**

(iii) 8, 9 and 25

Prime factors of 8 $= 2 \times 2 \times 2 = (2)^3$
Prime factors of 9 $= 3 \times 3 = (3)^2$
Prime factors of 25 $= 5 \times 5 = (5)^2$
$\therefore$ LCM (8, 9 and 25) $= (2)^3 \times (3)^2 \times (5)^2$
$= 8 \times 9 \times 25$
$= 1800$ **Ans.**
and HCF (8, 9 and 25) = 1 **Ans.**

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Solution : According to the question, the numbers are 306 and 657.

$\therefore$ $a = 306$
$b = 657$
and HCF = 9 [Given]

We know that

$$\text{L.C.M.} = \frac{a \times b}{\text{H.C.F.}}$$
$$= \frac{306 \times 657}{9}$$
$$= 34 \times 657$$
$$= 22338$$

Hence L.C.M. (306, 657) = 22338 **Ans.**

5. Check whether 6^n can end with the digit 0 for any natural number n.

Solution : Suppose that for any natural number n, $n \in$ N, 6^n ends with the digit 0. Hence 6^n will be divisible by 5.

But prime factors of $6 = 2 \times 3$
$\therefore$ The prime factors of $(6)^n$ will be
$$(6)^n = (2 \times 3)^n$$
i.e., it is clear that there is no place of 5 in the prime factors of 6^n.

By Fundamental theorem of Arithmatic we know that every composite number can be factorised as a product of prime numbers and this factorisation is unique, i.e., our hypothesis assured in the beginning is wrong. Hence there is no natural number n for which 6^n ends with the digit 0.

6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Solution : According to the question