From the session 2023-24, the textbooks are rationalised under the new National Education Policy 2020. This **Sanjiv Refresher** is completely based on the new rationalised textbook.

Sanjiv Refresher
Mathematics

For Basic and Standard Syllabus

Class - X

Main Features

- Strictly designed as per CBSE syllabus 2024-25 and the latest NCERT rationalised textbook edition.
- Detailed Explanation of Important Concepts with Solved Additional Examples.
- NCERT Textbook Exercises with Solutions.
- Exercises for Extra Practice.
- Objective Type Questions, Very Short Answer Type Questions and Case Study Based Questions as per the latest CBSE syllabus 2024-25.
- Important Questions from NCERT Exemplar.
- Inclusion of CBSE Board Examination Papers upto 2024 with solutions.

Publisher :

SANJIV PRAKASHAN

Jaipur

Price : ₹ 380/-

❑
Published by :
SANJIV PRAKASHAN
Dhamani Market, Chaura Rasta,
JAIPUR - 302003
email : sanjeevprakashanjaipur@gmail.com
website : www.sanjivprakashan.com

❑
Laser Type Setting :
Sanjiv Prakashan (DTP Department)
Jaipur

DISCLAIMER

Every effort has been made to remove the mistakes and omissions in this book. In case of any suggestion or any error found, send an email or post a letter to us at the following address :

Email : sanjeevprakashanjaipur@gmail.com
Postal Address : Publication Department
Sanjiv Prakashan,
Dhamani Market, Chaura Rasta,
Jaipur-302003

Your suggestions shall be taken care of in our next edition.

Though all the precautions have been taken in publishing this book yet for any mistake the author, publisher or printer is not liable. All disputes are subjected to Jaipur Jurisdiction only.

Contents

1

Real Numbers

1.1. Introduction

- **Algorithm :** Algorithm is a series of well defined steps which gives a procedure for solving a type of problem.
- **Lemma :** A lemma is a proven statement used for proving another statement.
- **Divisibility of Integers :** Any positive integer '*a*' can be divided by another positive integer '*b*' in such a way that it leaves remainder '*r*' that is smaller than '*b*'.
- **Product of Primes :** Every composite number can be expressed as a product of its primes.

1.2. The Fundamental Theorem of Arithmetic

"Every composite number can be expressed as a product of primes and this factorisation is unique apart from the order in which the prime factors occur."

For any two positive integers a and b, HCF $(a, b) \times$ LCM $(a, b) = a \times b$.

❖ **Important Points :**

(i) Euclid's division algorithm is not only useful in computing the HCF of large numbers but also important for the reason that this is one of more algorithms that were first of all used as a program in a computer.

(ii) Euclid's division lemma and Euclid's division algorithm are so closely interlinked that people often call Euclid's division lemma as Euclid's division algorithm.

(iii) Euclid's division lemma/algorithm is stated only for positive integers. However it can be applied for all integers (except zero, *i.e.*, $b \neq 0$).

EXAMPLES

Example 1. Using Fundamental Theorem of Arithmetic, find the LCM and HCF of 816 and 170. **[*CBSE 2010*]**

Solution : According to Fundamental Theorem of Arithmetic, every composite number can be expressed (or factorised) as a product of primes, which is unique, apart from the order in which the prime factors occur.

The prime factors of 816 and 170 gives us

$$816 = 2 \times 2 \times 2 \times 2 \times 3 \times 17 = 2^4 \times 3 \times 17$$

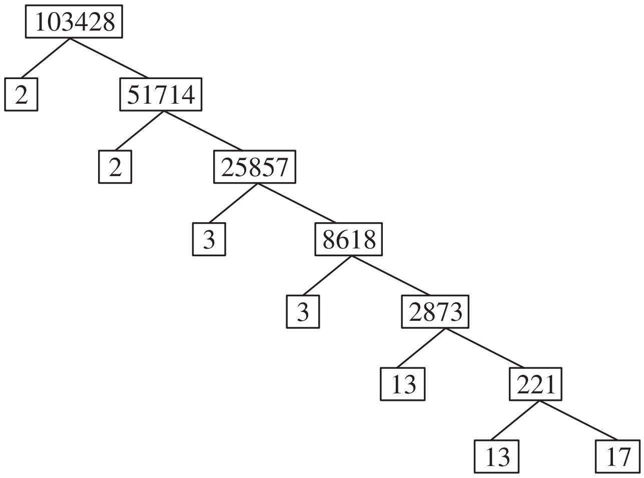
and $$170 = 2 \times 5 \times 17$$

$$\text{HCF}(816, 170) = 2 \times 17 = 34$$

and $$\text{LCM}(816, 170) = 2^4 \times 3 \times 5 \times 17 = 16 \times 15 \times 17 = 4080$$ **Ans.**

Example 2. Write the prime factorization of 103428 by using the factorisation tree.

Solution :



Therefore, the required factorization can be expressed as :

$$103428 = 2 \times 2 \times 3 \times 3 \times 13 \times 13 \times 17$$
$$= 2^2 \times 3^2 \times 13^2 \times 17^1$$ **Ans.**

Example 3. The HCF of two numbers a and b is 5 and their LCM is 200. Find the product ab. [*CBSE 2019*]

Solution : HCF of a and b = 5

LCM of a and b = 200

We know that

product of two numbers = HCF × LCM

$$a \times b = 5 \times 200$$

$\Rightarrow$ $$ab = 1{,}000$$ **Ans.**

Example 4. Find the least number which when divided by 12, 16 and 24 leaves the remainder 7 in each case. [*CBSE 2023*]

Solution : The least number which is divisible by 12, 16 and 24 is the LCM of 12, 16 and 24.

LCM of 12, 16 and 24 = 48

So the least number which when divided by 12, 16 and 24 leaves the remainder 7 in each case = 48 + 7 = 55 **Ans.**

Example 5. A circular field has a circumference of 360 km. Two cyclists Seema and Jyoti start together and can cycle at speeds of 12 km/hr and 15 km/hr respectively, round the circular field. After how many hours will they meet again at the starting point?

Solution : Speed of Seema = 12 km/hr

Number of hours taken by Seema to complete one round = $\frac{360}{12}$ = 30

Speed of Jyoti = 15 km/hr

Number of hours taken by Jyoti to complete one round = $\frac{360}{15}$ = 24

So, Seema and Jyoti complete one round in 30 hours and 24 hours respectively. Now, let us find the LCM of 30 and 24.

$$30 = 2 \times 3 \times 5, \quad 24 = 2^3 \times 3$$

Then LCM (30, 24) = $2^3 \times 3 \times 5 = 120$

Hence, Seema and Jyoti will meet each other again after 120 hours. **Ans.**

Example 6. If HCF (336, 54) = 6, find LCM (336, 54). [*CBSE 2019*]

Solution : According to the question, the numbers are 336, 54.

Given : $a = 336$, $b = 54$ and HCF = 6

We know that LCM $= \frac{a \times b}{\text{HCF}} = \frac{336 \times 54}{6} = 3024$

Hence, LCM (336, 54) = 3024 **Ans.**

Example 7. Explain why 2 × 3 × 5 + 5 and 5 × 7 × 11 + 7 × 5 are composite numbers. [*CBSE 2021*]

Solution : According to the question $2 \times 3 \times 5 + 5 = 5(2 \times 3 + 1)$

Since 5 is a factor of this number obtained, therefore it is a composite number.

Again, according to the question,

$$5 \times 7 \times 11 + 7 \times 5 = 5 \times 7\ (11 + 1)$$

Since 5 and 7 are factors of this number obtained, therefore it is a composite number.

Hence, both the given numbers are composite numbers. **Ans.**

NCERT Exercise 1.1

1. Express each number as a product of its prime factors :

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Solution : (i) Prime factors of 140 = $2 \times 70 = 2 \times 2 \times 35 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

(ii) Prime factors of 156 $= 2 \times 78 = 2 \times 2 \times 39$
$= 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

(iii) Prime factors of 3825 $= 3 \times 1275 = 3 \times 3 \times 425$
$= 3 \times 3 \times 5 \times 85 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$

(iv) Prime factors of 5005 $= 5 \times 1001 = 5 \times 7 \times 143 = 5 \times 7 \times 11 \times 13$

(v) Prime factors of 7429 $= 17 \times 437 = 17 \times 19 \times 23$ **Ans.**

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Solution : (i) 26 and 91 [*CBSE 2021*]

Prime factors of 26 = 2×13

Prime factors of 91 = 7×13

∴ LCM of 26 and 91 = $2 \times 7 \times 13 = 182$

and HCF of 26 and 91 = 13

Verification : HCF (26, 91) × LCM (26, 91) = $13 \times 182 = 13 \times 2 \times 91$
$= 26 \times 91$ = Product of given numbers **Ans.**

(ii) 510 and 92

Prime factors of 510 = $2 \times 255 = 2 \times 3 \times 85 = 2 \times 3 \times 5 \times 17$ .....(i)

and Prime factors of 92 = $2 \times 46 = 2 \times 2 \times 23 = 2^2 \times 23$ .....(ii)

LCM (510, 92) = $2^2 \times 3 \times 5 \times 17 \times 23 = 23460$

and HCF (510, 92) = 2

Verification : HCF (510, 92) × LCM (510, 92)

$= 2 \times 23460 = 2 \times 2^2 \times 3 \times 5 \times 17 \times 23$

$= 2 \times 3 \times 5 \times 17 \times 2^2 \times 23$

$= 510 \times 92$ = Product of given numbers **Ans.**

(iii) 336 and 54

Prime factors of 336 = $2 \times 168 = 2 \times 2 \times 84 = 2 \times 2 \times 2 \times 42$

$= 2 \times 2 \times 2 \times 2 \times 21 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$= 2^4 \times 3 \times 7$

Prime factors of 54 = $2 \times 27 = 2 \times 3 \times 9 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$

∴ HCF (336, 54) = $2 \times 3 = 6$

LCM (336, 54) = $2^4 \times 3^3 \times 7 = 3024$

Verification : HCF (336, 54) × LCM (336, 54)

$= 6 \times 3024 = 2 \times 3 \times 2^4 \times 3^3 \times 7 = 2^4 \times 3 \times 7 \times 2 \times 3^3$

$= 336 \times 54$ = Product of given numbers **Ans.**

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

Solution : (i) 12, 15 and 21

Prime factors of 12 = $2 \times 2 \times 3$

Prime factors of 15 = 3×5

Prime factors of 21 = 3×7

∴ LCM (12, 15 and 21) = $2^2 \times 3 \times 5 \times 7 = 420$

and HCF (12, 15 and 21) = 3 **Ans.**

(ii) 17, 23 and 29

Prime factors of 17 = 1×17

Prime factors of 23 = 1×23

Prime factors of 29 = 1×29

∴ LCM (17, 23 and 29) = $17 \times 23 \times 29 = 11339$

and HCF (17, 23 and 29) = 1 **Ans.**

(iii) 8, 9 and 25

Prime factors of 8 = $2 \times 2 \times 2 = (2)^3$

Prime factors of 9 = $3 \times 3 = (3)^2$

Prime factors of 25 = $5 \times 5 = (5)^2$

∴ LCM (8, 9 and 25) = $(2)^3 \times (3)^2 \times (5)^2 = 8 \times 9 \times 25 = 1800$

and HCF (8, 9 and 25) = 1 **Ans.**

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Solution : According to the question, the numbers are 306 and 657.

∴ $a = 306$, $b = 657$

and HCF = 9 (Given)

We know that LCM = $\frac{a \times b}{\text{HCF}} = \frac{306 \times 657}{9} = 34 \times 657 = 22338$

Hence, LCM (306, 657) = 22338 **Ans.**

5. Check whether 6^n can end with the digit 0 for any natural number n. **[*CBSE 2022*]**

Solution : Suppose that for any natural number n, $n \in N$, 6^n ends with the digit 0. Hence 6^n will be divisible by 5.

But prime factors of $6 = 2 \times 3$

$\therefore$ The prime factors of $(6)^n$ will be $(6)^n = (2 \times 3)^n$

i.e., it is clear that there is no place of 5 in the prime factors of 6^n.

By Fundamental Theorem of Arithmatic we know that every composite number can be factorised as a product of prime numbers and this factorisation is unique, *i.e.*, our hypothesis assured in the beginning is wrong. Hence there is no natural number n for which 6^n ends with the digit 0.

6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Solution : According to the question $7 \times 11 \times 13 + 13 = 13\,(7 \times 11 + 1)$

Since 13 is a factor of this number obtained, therefore it is a composite number.

Again, according to the question

$$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5\,(7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$$

This obtained number is also a composite number because it has also a factor 5.

Hence, both the given numbers are composite numbers. **Ans.**

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Solution : Time taken by Sonia to drive one round of the field = 18 minutes.

Time taken by Ravi to drive one round of the same field = 12 minutes.

In order to find after how much time will they meet again at the starting point, we shall have to find out the LCM of 18 and 12. Therefore,

prime factors of $18 = 2 \times 9 = 2 \times 3 \times 3 = 2 \times 3^2$

and prime factors of $12 = 2 \times 6 = 2 \times 2 \times 3 = 2^2 \times 3$

Taking the product of the greatest power of each prime factor of 18 and 12

$$\text{LCM}\,(18, 12) = 2^2 \times 3^2 = 4 \times 9 = 36$$

i.e., Sonia and Ravi will meet again at the starting point after 36 minutes. **Ans.**

PRACTICE EXERCISE 1.1

1. Determine the prime factorization of each of the following positive integers :

(i) 20570 (ii) 45470971

2. Give formal statement of the Fundamental Theorem of Arithmetic.

3. Find '*a*'