According to the Latest Syllabus

Sanjiv®

Mathematics

Class-11 (Part-I)

For the Students of Rajasthan Board of Secondary Education

By

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Preface

We are extremely pleased to present this book according to latest syllabus of NCERT. The book has been written in easy and simple language so that students may assimilate the subject easily. We hope that students will get benefitted from it and teachers will appreciate our efforts. In comparison to other books available in market, this book has many such features which make it a unique book :

1. Theoretical subject-material is given in adequate and accurate description along with pictures.
2. The latest syllabus of NCERT is followed thoroughly.
3. Complete solutions of all the questions given at the end of the chapter in the textbook are given in easy language.
4. Topic wise summary is also given in each chapter for the revision of the chapter.
5. In every chapter, all types of questions that can be asked in the exam (Objective, Fill in the blanks, Very short, Short, Numerical and Long answer type questions) are given.
6. At the end of every chapter, multiple choice questions asked in various competitive exams are also given with solutions.

Valuable suggestions received from subject experts, teachers and students have also been given appropriate place in the book.

We wholeheartedly bow to the Almighty God, whose continuous inspiration and blessings have made the writing of this book possible.

We express our heartfelt gratitude to the publisher – Mr. Pradeep Mittal and Manoj Mittal of Sanjiv Prakashan, all their staff, laser type center and printer for publishing this book in an attractive format on time and making it reach the hands of the students.

Although utmost care has been taken in publishing the book, human errors are still possible, hence, valuable suggestions are always welcome to make the book more useful.

In anticipation of cooperation!

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CHAPTER 1

SETS

Chapter Overview

1.1 Introduction

The concept of set serves as a fundamental part of the present day mathematics. Today this concept is being used in almost every branch of mathematics. Sets are used to define the concepts of relations and functions. The study of geometry, sequences, probability, etc. requires the knowledge of sets.

The theory of sets was developed by German mathematician Georg Cantor (1845-1918).

1.2 Sets and their Representations

Definition : "A set is a well defined collection of objects." Here, the word well defined means that there should be no uncertainty or difference as to whether the objects are included in the collection or not.

With the help of set theory, any fact can be expressed in a comprehensive manner and language can also be reduced.

Sets are usually denoted by capital letters A, B, C, X, Y, Z etc. The objects that make up a set are called elements of the set. These elements are represented by small letters a, b, c, x, y, z etc. of x is an element of a set A, we say that "x belongs to A" the Greek symbol $\in$ (epsilon) is used to denote the phrase 'belongs to'. Thus we write $\boxed{x \in A}$. If 'x' is not an element of set A, we write $\boxed{x \notin A}$ and read "x does not belong to A".

Note : The following symbols are helpful in expressing our point in a subtle manner.

$\cup$: or
$\forall$: for all

$\cap$: and
$\in$: belongs to
$\notin$: does not belongs to
$\subset$; is a proper subset
$\not\subset$: is not a subset

Few more examples of sets used particularly in mathematics :

N : the set of all natural numbers
W : the set of all whole numbers
Z : the set of all integers
Z^+ : the set of all positive integers
Z^- : the set of all negative integers
R : the set of all real numbers
R^+ : the set of all positive real numbers
R^- : the set of all negative real numbers
Q : the set of all rational numbers
Q^+ : the set of all positive rational numbers
Q^- : the set of all negative rational numbers
C : the set of all complex numbers

Some other examples of set, which are used in maths :
(i) Even natural numbers less than 10, i.e. 2, 4, 6, 8
(ii) Vowel of English elphabet i.e. *a, e, i, o, u*
(iii) Rivers of India
(iv) Solution of $x^2 + 7x + 12 = 0$ is – 3 and – 4.
(v) Different types of triangles.
(vi) Prime factorisation of 24 i.e. 2, 2, 2, 3

Here, it is clear that the collection of objects is well defined. We can decide for an object with certainty whether a particular object is included in a given collection or not.

Representation of Sets : There are two methods of representing a set : (i) Roster of tabular form, (ii) Set-builder form.

(1) Roster or Tabular Form : In this form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within braces { }. For example :

(i) The set of all the letters of the word RAJASTHAN = {R, A, J, S, T, H, N}

(ii) The set of all vowels in the English alphabet = {*a*, *e*, *i*, *o*, *u*}

(iii) The set of all natural numbers which divide 24 = {1, 2, 3, 4, 6, 8, 12, 24}

(iv) The set of all odd positive integers which are less than 15 = {1, 3, 5, 7, 9, 11, 13}

Memory Point

(a) The order in which the objects are listed in tabular form is meaningless. For example : the set of all vowels in the English alphabet can be represented by {i, a, e, u, o}.

(b) While writing a set in a tabular form, generally no element can be repeated. All objects are different. For example : The set of letters forming the word 'MATHEMATICS' is {M, A, T, H, E, I, C, S}.

(2) Rule Form or Set-builder Form : This method is used when the number of elements in the set is very large, In this method, first of all, all the elements of the set are expressed by some variable quantity x. Then we put the symbol (:) or (/) by writing x in the braces { } and after this, write the property which its elements satisfy. For example :

(i) Set of all citizens of Bharat
A = {x : x is a citizen of Bharat}

(ii) Set of all natural numbers which divide 24
B = {x : x is a natural number which divide 24}

(iii) Set of all even natural numbers
E = {z : z is an even natural number}

(iv) A = {x : x is a natural number and $3 < x < 10$}. It is read as the set of all x, where x is a natural number and x lies between 3 and 10. Hence
A = {4, 5, 6, 7, 8, 9}

(v) Taking even natural numbers less than 10, the set-builder form is expressed as follows :
A = {x : x is a even natural number which is less than 10}

Example 1. Write the solution of the equation $x^2 + x - 2 = 0$ in roster form. (NCERT)

Sol. The given equation can be written as
$(x - 1)(x + 2) = 0$ i.e., $x = 1, -2$

Therefore, the solution set of the given equation can be written in roster form {1, – 2}.

Example 2. Write the set {x : x is a positive integer and $x^2 < 40$} in the roster form. (NCERT)

Sol. The required numbers are 1, 2, 3, 4, 5, 6. So, the given set in the roster form is {1, 2, 3, 4, 5, 6}.

Example 3. Set {x : x is even natural number divisible by 3 and less than 21}.

Sol. The required even natural numbers are 6, 12, 18.
Hence the roster form of required number is {6, 12, 18}.

Example 4. Write the set A = {1, 4, 9, 16, 25, ...} in set-builder form. (NCERT)

Sol. We may write the set A as
A = {x : x is the square of a natural number}
Alternatively, we can write
A = {x : $x = n^2$, where $n \in$ N}.

Example 5. Write the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ in the set-builder form. (NCERT)

Sol. We see that each member in the given set has the numerator one less than the denominator. Also, the numerator begin from 1 and do not exceed 6. Hence, in the set-builder form the given set is :

{x : $x = \frac{n}{n+1}$, where n is a natural number and $1 \le n \le 6$}

Example 6. Write the following sets in the set-builder form :

(i) A = {5, 10, 15, ...}

(ii) B = {14, 21, 28, 35, 42, ..., 98}

(iii) C = $\left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, ...\right\}$

(iv) D = {– 1, 1}

Sol. (i) A = {x : x is a natural number which is multiple of 5}

(ii) B = {x : x is a natural number which is multiple of 7 and $7 < x < 100$}

(iii) C = $\left\{\frac{1}{n^2} : n \in N\right\}$

(iv) D = {x : x is an odd integer and $|x| < 2$}

Example 7. Write the Roster form of the set A = {x : $3x - 4 < 6$, $x \in N$}

Sol. $3x - 4 < 6$

$\Rightarrow 3x < 10$

$\Rightarrow x = \frac{10}{3}$ and $x \in N$

$\Rightarrow$ A = {1, 2, 3}

Example 8. Match each of the set on the left described in the roster form with the same set on the right described in the set-builder form : (NCERT)

(i) {P, R, I, N, C, A, L}	**(a) {x : x is a positive integer and is a divisor of 18}**
(ii) {0}	**(b) {x : x is an integer and $x^2 - 9 = 0$}**
(iii) {1, 2, 3, 6, 9, 18}	**(c) {x : x is an integer and $x + 1 = 1$}**
(iv) {3, –3}	**(d) {x : x is a letter of the word PRINCIPAL}**

Sol. Since in (d), there are 9 letters in the word PRINCIPAL and two letters P and I are repeated, so, (i) matches (d). Similarly, (ii) matches (c) as $x + 1 = 1$ implies $x = 0$. Also, 1, 2, 3, 6, 9, 18 are the divisors of 18 and so (iii) matches (a). Finally, $x^2 - 9 = 0$ implies $x = 3, -3$ and so (iv) matches (b).

Exercise 1.1

1. Which of the following are sets? Justify your answer.

(i) The collection of all the months of a year beginning with the letter J.

(ii) The collection of ten most talented writers of India.

(iii) A team of eleven best-cricket batsmen of the world.

(iv) The collection of all boys in your class.

(v) The collection of all natural numbers less than 100.

(vi) A collection of novels written by the writer Munshi Prem Chand.

(vii) The collection of all even integers.

(viii) The collection of questions in this Chapter.

(ix) A collection of most dangerous animals of the world.

Sol. (i) This is a set because this is well defined collection. The collection of all the months of a year beginning with the letter J are January, June and July.

(ii) This is not a set because the collection of ten most talented writers of India (Bharat) is not well defined.

(iii) This is not a set because the collection of eleven best-cricket batsmen is not well defined.

(iv) This collection is a set because the collection of all boys in my class is well defined.

(v) This is a set because the collection of all natural numbers less than 100 are 1, 2, 3, ... 99 which is well defined.

(vi) This is a set because the collection of novels written by the writer Munshi Prem Chand is well defined.

(vii) This is a set because the collection of all even integers i.e. {........–6, –4, –2, 0, 2, 4, 6, ...} is well defined.

(viii) This is a set because the collection of questions in this Chapter is well defined.

(ix) This is not a set because collection of most dangerous animals of the world is not well defined.

2. Let A = {1, 2, 3, 4, 5, 6}. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces :

(i) 5. . .A (ii) 8. . .A (iii) 0. . .A

(iv) 4. . .A (v) 2. . .A (vi) 10. . .A

Sol. (i) $5 \in A$ (ii) $8 \notin A$

(iii) $0 \notin A$ (iv) $4 \in A$

(v) $2 \in A$ (vi) $10 \notin A$

3. Write the following sets in roster form :

(i) A = {x : x is an integer and $-3 < x < 7$}

(ii) B = {x : x is a natural number less than 6}

(iii) C = {x : x is a two-digit natural number such that the sum of its digits is 8}

(iv) D = {x : x is a prime number which is divisor of 60}

(v) E = The set of all letters in the word TRIGONOMETRY

(vi) F = The set of all letters in the word BETTER

Sol. (i) A = {–2, –1, 0, 1, 2, 3, 4, 5, 6}

(ii) B = {1, 2, 3, 4, 5}

(iii) C = {17, 26, 35, 44, 53, 62, 71, 80}

(iv) D = {2, 3, 5} Since 2, 3, 5 are prime numbers and divisors of 60.

(v) E = {T, R, I, G, O, N, M, E, Y} ($\because$ Letters are written only once in a set)

(vi) F = {B, E, T, R} ($\because$ Letters are written only once in a set)

4. Write the following sets in the set-builder form :

(i) {3, 6, 9, 12} (ii) {2, 4, 8, 16, 32}
(iii) {5, 25, 125, 625} (iv) {2, 4, 6, . . .}
(v) {1, 4, 9, . . .,100}

Sol. (i) A = {x : x is a natural number and multiple of 3} i.e. A = {x : $x = 3n$, $n \in$ N and $1 \le n \le 4$}

(ii) B = {x : $x = 2^n$, $n \in$ N and $1 \le n \le 5$}

(iii) C = {x : $x = 5^n$, $n \in$ N and $1 \le n \le 4$}

(iv) D = {x : x is an even natural number}

(v) E = {x : $x = n^2$, $n \in$ N and $1 \le n \le 10$}

5. List all the elements of the following sets :

(i) A = {x : x is an odd natural number}

(ii) B = {x : x is an integer, $-\frac{1}{2} < x < \frac{9}{2}$ }

(iii) C = {x : x is an integer, $x^2 \le 4$}

(iv) D = {x : x is a letter in the word "LOYAL"}

(v) E = {x : x is a month of a year not having 31 days}

(vi) F = {x : x is a consonant in the English alphabet which precedes k}.

Sol. (i) A = {1, 3, 5, ...}

(ii) B = {0, 1, 2, 3, 4}

(iii) C = {–2, –1, 0, 1, 2}

(iv) D = {L, O, Y, A}

(v) E = {February, April, June, September, November}

(vi) F = {b, c, d, f, g, h, j}

6. Match each of the set on the left in the roster form with the same set on the right described in set-builder form :

(i) {1, 2, 3, 6}	(a) {x : x is a prime number and a divisor of 6}
(ii) {2, 3}	(b) {x : x is an odd natural number less than 10}
(iii) {M,A,T,H,E,I,C,S}	(c) {x : x is natural number and divisor of 6}
(iv) {1, 3, 5, 7, 9}	(d) {x : x is a letter of the word MATHEMATICS}

Sol. (i) $\longleftrightarrow$ (c) (ii) $\longleftrightarrow$ (a)
(iii) $\longleftrightarrow$ (d) (iv) $\longleftrightarrow$ (b)

1.3 Types of Sets

(1) Singleton Set : The set which has one and only one element is called singleton set. Thus the set {a} is a singleton set having only one element a. For example **:**

(i) A = {x : $9 < x < 11$ and $x \in$ N} = {10} is a singleton set.

(ii) A = {x : $5x = 7$, $x \in$ Q} is a singleton set, $\because$ $x = \frac{7}{5}$ is solution of equation $5x = 7$.

(2) Empty Set or Null Set or Void Set : A set which does not contain any element is called the empty set or the null set or the void set. The empty set is denoted by the symbol ϕ or { }. For example **:**

(i) A = {x : $2 < x < 3$, $x \in$ N} = { } or ϕ

(ii) A = set of the point of intersection of two parallel lines = { } or ϕ

(iii) A = {x : x is real and $x^2 + 1 = 0$} is an empty set because $x^2 + 1 = 0$ does not have any solution.

Memory Point

✡ Set {0} is not empty set because it has element 0.
✡ Set {ϕ} is not empty set because it has element ϕ.

(3) Finite Set : A set which is empty or consists of a definite number of elements is called finite.

Example : (i) A = {x : $x \le 20$, $x \in$ N}
= {1, 2, 3, ..., 20} is a finite set.

(ii) The set of a prime factors of 60 is a finite set.

(iii) The set of months in a year is infinite set.

(4) Infinite Set : A set in which the number of elements present is unlimited or infinite is called an infinite set.

Example : (i) A = {x : $x > 10$, $x \in$ N}
= {11, 12, 13, 14, ... ∞} is an infinite set.

(ii) B = {x : $x = 3^n$, n is a positive integer including 0} is an infinite set.

(iii) Set of all natural numbers N = {1, 2, 3, 4, 5, ...}

(iv) Set of all whole numbers W = {0, 1, 2, 3, 4, 5, ...}

1.4 Order of a Finite Set

The number of different elements present in a finite set 'S' is called the order of the set. This is represented by n (S) or O (S).

Note that the order of an infinite set cannot be defined.

For example : (i) If S = {2, 4, 6, 8, 10, 12} then n (S) = 6.

(ii) Order of an empty set is zero i.e. n { } = 0, but n {0} = 1.

1.5 Equivalent Sets

Two sets A and B are said to be equivalent if the number of element in set A is equal to number of elements in set B i.e. n (A) = n (B). It is represented by A ~ B and read as 'A is equivalent to B'.

For example : If A = {1, 2, 3}, B = {a, b, c}, C = {x, y, z} then n (A) = n (B) = n (C) = 3. Hence A, B and C are equivalent sets which are represented as A ~ B ~ C.

1.6 Equal Sets

Two sets A and B are said to be *equal* if they have exactly the same elements and we write A = B. Otherwise, the sets are said to be *unequal* and we write A ≠ B. Hence, if $\forall x \in A \Rightarrow x \in B$ and $\forall y \in B \Rightarrow y \in A$, then the sets A and B are equal. i.e. A = B.

For example : (i) {4, 6, 8} = {6, 8, 4}
[The order is not important in set]

(ii) {3, 6, 12} = {3, 6, 6, 12, 12, 3}
[Repetition of elements is meaningless]

(iii) {2, 4, 8, ...} = $\{x : x = 2^n, n \in N]$

Example 9. State which of the following sets are finite or infinite :

(i) $\{x : x \in N$ and $(x - 1)(x - 2) = 0\}$
(ii) $\{x : x \in N$ and $x^2 = 4\}$
(iii) $\{x : x \in N$ and $2x - 1 = 0\}$
(iv) $\{x : x \in N$ and x is prime$\}$
(v) $\{x : x \in N$ and x is odd$\}$ (NCERT)

Sol. (i) Given set = {1, 2}. Hence, it is finite.
(ii) Given set = {2}. Hence, it is finite.
(iii) Given set = ϕ. Hence, it is finite.
(iv) The given set is the set of all prime numbers and since set of prime numbers is infinite. Hence, the given set is infinite.
(v) Since there are infinite number of odd numbers, hence, the given set is infinite.

Example 10. Find the pairs of equal sets, if any, give reasons :

A = {0}, B = $\{x : x > 15$ and $x < 5\}$
C = $\{x : x - 5 = 0\}$ D = $\{x : x^2 = 25\}$
E = $\{x : x$ is an integral positive root of the equation $x^2 - 2x - 15 = 0\}$ (NCERT)

Sol. Since 0 ∈ A and 0 does not belong to any of the sets B, C, D and E, it follows that, A ≠ B, A ≠ C, A ≠ D, A ≠ E.

Since B = ϕ but none of the other sets are empty. Therefore B ≠ C, B ≠ D and B ≠ E. Also C = {5} but –5 ∈ D hence, C ≠ D.

Since E = {5}, C = E. Further, D = {– 5, 5} and E = {5}, we find that, D ≠ E. Thus, the only pair of equal sets is C and E.

Example 11. Which of the following pairs of sets are equal? Justify your answer.

(i) X, the set of letters in "ALLOY" and B, the set of letter in "LOYAL".

(ii) A = $\{n : n \in Z$ and $n^2 \le 4\}$ and B = $\{x : x \in R$ and $x^2 - 3x + 2 = 0\}$ (NCERT)

Sol. (i) We have, X = {A, L, L, O, Y}, B = {L, O, Y, A, L}. Then X and B are equal sets as repetition of elements in a set do not change a set. Thus, X = {A, L, O, Y} = B

(ii) A = {–2, –1, 0, 1, 2}, B = {1, 2}. Since 0 ∈ A and 0 ∉ B, A and B are not equal sets.

Example 12. State which of the following sets are finite or infinite :

(i) The set of points equidistant from two intersecting lines.
(ii) The set of lines perpendicular to given line.
(iii) The set of points equidistant from two given pionts.
(iv) The set of even natural numbers.
(v) The set of natural number which are factors of 36.
(vi) The empty set ϕ.
(vii) The set of multiples of 5.
(viii) The set of lines parallel to a given line.

Sol. (i) Infinite (ii) Infinite (iii) Infinite (iv) Infinite (v) Finite (vi) Finite (vii) Infinite (viii) Infinite.

Example 13. Which of the following sets are equal?

(i) {1, 2, 3}
(ii) Set of the factors of 6.
(iii) The set of natural numbers, less than 4.
(iv) {1, 2, 3, 4}

Sol. (i) {1, 2, 3}
(ii) {1, 2, 3} = Set of factors of 6.
(iii) Set of natural numbers which are less than 4 = {1, 2, 3}
(iv) {1, 2, 3, 4}

From above, (i) = (ii) = (iii) ≠ (iv) i.e. (i), (ii) and (iii) are equal sets.

Exercise 1.2

1. **Which of the following are examples of the null set :**
   (i) Set of odd natural numbers divisible by 2
   (ii) Set of even prime numbers
   (iii) $\{ x : x$ is a natural numbers, $x < 5$ and $x > 7 \}$
   (iv) $\{ y : y$ is a point common to any two parallel lines$\}$

Sol. (i) There does not exist any odd natural number which is divisible by 2, hence this is an empty set ϕ.
(ii) We know that 2 is the only even prime number, therefore, this is not an empty set.
(iii) There does not exist any natural number which is less than 5 and greater than 7. Therefore, this is an empty set.
(iv) Parallel lines never intersect each other. Thus there is no common point to any two parallel lines. Hence, this is an empty set ϕ.

2. **Which of the following sets are finite or infinite :**
   (i) The set of months of a year
   (ii) {1, 2, 3, . . .}
   (iii) {1, 2, 3, . . .99, 100}