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Sanjiv Refresher

MATHEMATICS

Based on Latest NCERT Text Book

For Class-9

Price : ₹ 340/-

Publisher :

SANJIV PRAKASHAN

Jaipur

- Published by :
  Sanjiv Prakashan
  Dhamani Market, Chaura Rasta,
  Jaipur-302003
  email : sanjeevprakashanjaipur@gmail.com
  website : www.sanjivprakashan.com

- Laser Type Setting :
  Sanjiv Prakashan (D.T.P. Department),
  Jaipur

- Printers :
  Manohar Art Printer, Jaipur
  ****

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Contents

Mathematics—Class IX

1. NUMBER SYSTEMS

CHAPTER SUMMARY

1. **Natural Numbers :** Numbers from one onwards are known as natural numbers. There are infinitely many natural numbers. The collection of all natural numbers is represented by the symbol N. i.e.,
   $N = \{1, 2, 3, 4, 5, ...\}$
2. **Whole Numbers :** If we include zero (0) in the collection of all natural numbers to get 0, 1, 2, 3, 4, 5, ..., these numbers are called whole numbers. This new collection of whole numbers is denoted by W and is written as :
   $W = \{0, 1, 2, 3, 4, 5, ...\}$
3. **Integers :** If we include negative numbers in the set of whole numbers, we get a new set of numbers which is known as set of integers. The set of integers is denoted by I or Z. Thus
   $I/Z = \{....., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...\}$
4. **Rational Numbers :** Numbers of the form , where p and q are integers and $q \neq 0$ are known as rational numbers. The collection of such numbers is called set of rational numbers. It is denoted by Q and is written as :
   $Q = \left\{\frac{p}{q}\right.$, where p and q are integers and $q \neq 0\}$
   Numbers like $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{2}{3}, \frac{-2}{5}$.... etc. are known as rational numbers.
5. A rational number may be positive, negative or zero. A rational number $\frac{p}{q}$ is positive, if both p and q have the same sign while it is negative if p and q have opposite signs.
6. Every integer is a rational number but every rational number may not be an integer. The decimal representation of a rational number is either terminating or non-terminating recurring.
7. **Terminating decimal representation :** For a rational number of the form $\frac{p}{q}$ (p, q are integers and $q \neq 0$), when we divide p by q, the long division terminates after a finite number of steps and the remainder becomes zero. Such type of decimal expansions are known as terminating decimal expansions.
   Example : $\frac{3}{8} = 0.375, \frac{1}{4} = 0.25, \frac{456}{125} = 3.648$ etc.
8. **Non-terminating decimal representation :** There are rational numbers such that when we try to express them in decimal form by division method, we find that no matter how long we divide there is always a remainder. In other words, the division process never comes to an end. This is due to the reason that in the

division process the remainder starts repeating after a certain number of steps. Such decimals are called non-terminating repeating decimals or non-terminating recurring decimals.

Example : $\frac{8}{9} = 0.88888.....$, $\frac{6}{7} = 0.857142857142.....$ etc.

9. The set of rational numbers includes natural numbers, whole numbers and integers. Zero (0) is also a rational number.

10. If we multiply the numerator and denominator of a rational number by the same number, then value of the rational number does not change.

11. To find a rational number between any two given numbers :

Rational number between two numbers = $\frac{\text{First number + Second number}}{2}$

12. Irrational Numbers : A number 's' is called irrational if it cannot be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
Example : $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$ etc. are irrational numbers.

13. Between two given rational numbers, there are infinitely many rational numbers.

14. The decimal expansion of an irrational number is non-terminating non-recurring. Moreover, a number whose decimal expansion is non-terminating non-recurring is irrational.

15. A number is irrational if it has a non-terminating and non-repeating decimal representation. For example, π and e are non-terminating non-repeating decimals, so they are irrational numbers.

16. Real Numbers : Collection of rational as well as irrational numbers is termed as the collection of real numbers. Thus, a real number is either a rational real number or an irrational real number. The collection of real numbers is denoted by R. Corresponding to each real number in R, there is a point on the number line and conversely, corresponding to each point on the number line, there is a real number in R. Since all rational numbers and irrational numbers can be represented on the number line, so we call the number line as real number line.

17. Any real number is represented on the number line in its unique point while every point on the number line represents a unique real number.

18. For any real number 'a'
$|a| = a$ if $a \geq 0$
and $|a| = -a$ if $a < 0$

19. $|a|$, is known as absolute value of a
$|a| = |-a| = a$ if a is a positive real number.

20. If 'r' is rational and 's' is irrational, then $(r+s)$ and $(r-s)$ are irrational numbers and if $r \neq 0$ then rs and r/s are irrational numbers.

21. If a and b are two rational numbers which are not perfect squares then irrational numbers $(\sqrt{a}+\sqrt{b})$ and $(\sqrt{a}-\sqrt{b})$ are known as conjugate of each other and
(i) product of two irrational conjugate numbers is always a rational number.
(ii) a binomial quadratic irrational number can be rationalised very easily by multiplying it by its conjugate.

22. For positive real numbers a and b, the following identities hold :

(i) $\sqrt{ab} = \sqrt{a}\sqrt{b}$ (ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(iii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a-b$ (iv) $(a+\sqrt{b})(a-\sqrt{b}) = a^2-b$

(v) $(\sqrt{a}+\sqrt{b})^2 = a+2\sqrt{ab}+a$

(vi) $(\sqrt{a}+\sqrt{b})(\sqrt{c}-\sqrt{d}) = \sqrt{ac}-\sqrt{ad}+\sqrt{bc}-\sqrt{bd}$

(vii) $\frac{1}{a+\sqrt{b}} = \frac{a-\sqrt{b}}{a^2-b}$ (viii) $\frac{1}{a+b\sqrt{x}} = \frac{a-b\sqrt{x}}{a^2-b^2x}$, where x is a real number.

(ix) $\frac{1}{\sqrt{x}+\sqrt{y}} = \frac{\sqrt{x}-\sqrt{y}}{x-y}$, where x and y are real numbers.

23. To rationalise the denominator of $\frac{1}{\sqrt{a}+b}$, we multiply this by $\frac{\sqrt{a}-b}{\sqrt{a}-b}$, where a and b are integers.

24. Let $a > 0$ be a real number and p and q be rational numbers. Then :

(i) $a^p \,.\, a^q = a^{p+q}$ (ii) $(a^p)^q = a^{pq}$

(iii) $\frac{a^p}{a^q} = a^{p-q}, p > q$ (iv) $a^p \,.\, b^p = (ab)^p$

25. $\frac{0}{0}$ and $(0)^0$ are undefined.

26. $a^0 = 1$ if $a \neq 0$

27. Let $a > 0$ be a real number. Let m and n be integers such that m and n have no common factors other than 1, and $n > 0$, then :

(i) $a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$ (ii) $\sqrt[n]{a} = \sqrt[n\times p]{a^p}$

TEXTBOOK QUESTIONS

Exercise 1.1

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Solution : Yes, 0 is a rational number.

0 can be written in any of the following forms :

$\frac{0}{1}, \frac{0}{-2}, \frac{0}{3}, \frac{0}{-4}$ and so on.

Thus, 0 can be written as $\frac{p}{q}$, where $p = 0$ and q is any non-zero integer.

Hence, 0 is a rational number.

2. Find six rational numbers between 3 and 4.

Solution : We know that between two rational numbers a and b, such that $a < b$.

There is always a rational number $\frac{a+b}{2}$, where $a < \frac{a+b}{2} < b$.

A rational number between 3 and 4 is $\frac{1}{2}(3 + 4)$, *i.e.* $\frac{7}{2}$.

$\therefore \quad 3 < \frac{7}{2} < 4.$

Now, a rational number between 3 and $\frac{7}{2}$ is :

$$\frac{1}{2}\left(3+\frac{7}{2}\right) = \frac{1}{2}\times\frac{6+7}{2} = \frac{13}{4}.$$

A rational number between $\frac{7}{2}$ and 4 is :

$$\frac{1}{2}\left(\frac{7}{2}+4\right) = \frac{1}{2}\times\frac{7+8}{2} = \frac{15}{4}.$$

$\therefore \quad 3 < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} < 4$

Further, a rational number between 3 and $\frac{13}{4}$ is :

$$\frac{1}{2}\left(3+\frac{13}{4}\right) = \frac{1}{2}\times\frac{12+13}{4} = \frac{25}{8}.$$

A rational number between $\frac{15}{4}$ and 4 is :

$$\frac{1}{2}\left(\frac{15}{4}+4\right) = \frac{1}{2}\times\frac{15+16}{4} = \frac{31}{8}.$$

A rational number between $\frac{31}{8}$ and 4 is :

$$\frac{1}{2}\left(\frac{31}{8}+4\right) = \frac{1}{2}\times\frac{31+32}{8} = \frac{63}{16}.$$

$\therefore\ 3 < \frac{25}{8} < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} > \frac{31}{8} < \frac{63}{16} < 4$

Hence, six rational numbers between 3 and 4 are :

$\frac{25}{8}, \frac{13}{4}, \frac{7}{2}, \frac{15}{4}, \frac{31}{8}$ and $\frac{63}{16}$. **Ans.**

Aliter : There can be infinitely many rational numbers between the numbers 3 and 4, one way is to multiply and divide by 6 + 1 = 7, to get an equivalent fraction like :

$$3 \times \frac{7}{7} = \frac{21}{7}.$$

$$4 \times \frac{7}{7} = \frac{28}{7}.$$

We know that 21 < 22 < 23 < 24 < 25 < 26 < 27 < 28

$\Rightarrow \quad \frac{21}{7} < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < \frac{28}{7}$

Hence, six rational numbers between 3 and 4 are :

$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}$ and $\frac{27}{7}$. **Ans.**

3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Solution : Since, we want 5 rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$, so we write :