According to the Latest NCERT Syllabus

Sanjiv®

Mathematics

Class-12 (Part-I)

For the Students of Rajasthan Board of Secondary Education

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Preface

We are extremely pleased to present this book according to latest syllabus of NCERT. The book has been written in easy and simple language so that students may assimilate the subject easily. We hope that students will get benefitted from it and teachers will appreciate our efforts. In comparison to other books available in market, this book has many such features which make it a unique book :

1. Theoretical subject-material is given in adequate and accurate description along with pictures.
2. The latest syllabus of NCERT is followed thoroughly.
3. Complete solutions of all the questions given at the end of the chapter in the textbook are given in easy language.
4. Topic wise summary is also given in each chapter for the revision of the chapter.
5. In every chapter, all types of questions that can be asked in the exam (Objective, Fill in the blanks, Very short, Short, Numerical and Long answer type questions) are given.
6. At the end of every chapter, multiple choice questions asked in various competitive exams are also given with solutions.

Valuable suggestions received from subject experts, teachers and students have also been given appropriate place in the book.

We wholeheartedly bow to the Almighty God, whose continuous inspiration and blessings have made the writing of this book possible.

We express our heartfelt gratitude to the publisher – Mr. Pradeep Mittal and Manoj Mittal of Sanjiv Prakashan, all their staff, laser type center and printer for publishing this book in an attractive format on time and making it reach the hands of the students.

Although utmost care has been taken in publishing the book, human errors are still possible, hence, valuable suggestions are always welcome to make the book more useful.

In anticipation of cooperation!

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CHAPTER 1

RELATIONS AND FUNCTIONS

Chapter Overview

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1.1 Introduction

❖ In class XI we have studied set, subset, Cartesian product, relation, domain and co-domain, range of relation, function, domain, co-domain and range of function in detail. In this chapter we shall recall the above definitions and then study the types of functions in detail.

1.2 Ordered Pair

❖ Generally on changing the order of the elements of the set, no change occurs in the set. For example, $\{1, 2\} = \{2, 1\}$ but if the order of the elements of any set has importance, then such a set is called ordered set. Similarly if in the set $\{a, b\}$ of two elements, a is assigned the first place and b is assigned the second place, then this set is called an ordered pair and it is expressed by the symbol (a, b). Here $(a, b) \neq (b, a)$.

From definition it is clear that

$(a, b) = (c, d) \Leftrightarrow a = c$ and $b = d$

❖ If in any ordered set, the number of element be n, then such a set is called ordered n-tuple and it is expressed by $(a_1, a_2, ..., a_n)$. For example : In two dimensional coordinates (x, y) and three dimensional cordinates (x, y, z), order has its importance.

1.3 Cartesian Product of two sets

❖ The Cartesian product of two sets A and B is the set of some ordered pair where first element a is the element of set A and second element b is the element of set B. This product is expressed by the symbol A × B. Hence,

$A \times B = \{(a, b) \mid a \in A, b \in B\}$

❖ By definition it is clear that $A \times B \neq B \times A$ until A and B are equal.

☞ **Example.** If A = {p, q, r} and B = {x, y} then
A × B = {(p, x), (p, y), (q, x), (q, y), (r, x), (r, y)}
B × A = {(x, p), (y, p), (x, q), (y, q), (x, r), (y, r)}

❖ **Remarks :**

(i) If $A = \phi$ or $B = \phi$, then $A \times B = \phi$ Here ϕ is the null set.

(ii) If $A = \phi$ and $B = \phi$, then $A \times B = \phi$

(iii) If the number of elements in the set A is m and the number of elements in the set B is n, then there will be $m \times n$ elements in A × B. Hence, the number of its non-empty subsets will be $2^{mn} - 1$.

(iv) If A and B are non-empty sets and either one or both of them are infinite sets and the number of elements in A × B will be infinite, *i.e.*, A × B also will be an infinite set.

(v) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(vi) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(vii) $A \times (B - C) = (A \times B) - (A \times C)$

(viii) $A \subseteq B \Rightarrow A \times C \subseteq B \times C$

(ix) $A \subseteq B, C \subseteq D \Rightarrow (A \times C) \subseteq (B \times D)$

1.4 Relation

❖ The relation R defined from set A into set B is a subset of A × B, *i.e.*, $R \subseteq A \times B$.

$R = \{(x, y) \mid xRy, x \in A, y \in B\}$

❖ If a and b are related by relation R then this fact can be written as follows :

a R b or $(a,b) \in R$

❖ If a and b are not related by relation R, then we can write this fact as follows :

$a \not R b$ or $(a, b) \notin R$

☞ **Example 1.** If A = {3, 5, 7, 9,}, B = {4, 6, 8, 10} and P(x,y) = x is smaller than y, then

R = {A, B, P (x, y)}

is a relation from A into B. Under this relation 3R4, 3R6, 3R8, 3R10, 5R6, 5R8, 5R10, 7R8, 7R10, 9R10, but $5\not R4$, $7\not R4$, $7\not R6$, $9\not R4$, $9\not R6$, $9\not R8$.

We can express it also as follows :

$(3,4) \in R$, $(3,6) \in R$, $(3,8) \in R$, $(3,10) \in R$, $(5,6) \in R$, $(5,8) \in R$, $(5,10) \in R$, $(7,8) \in R$, $(7,10) \in R$, $(9,10) \in R$, but $(5,4) \notin R$, $(7,4) \notin R$, $(7,6) \notin R$, $(9,4) \notin R$, $(9,6) \notin R$, $(9,8) \notin R$ etc. Then R = {(3, 4), (3, 6), (3, 8), (3, 10), (5, 6), (5, 8), (5, 10), (7, 8), (7, 10), (9, 10)}

☞ **Example 2.** If A = {2, 3, 4}, B = {3, 6, 8} and P(x,y) = x is divisor of y, then

R = {A, B, P(x,y)}

is a relation from A into B. Under this relation 2R6, 2R8, 3R3, 3R6, 4R8 but $2\not R3$, $3\not R8$, $4\not R3$, $4\not R6$, *i.e.* $(2, 6) \in R$, $(2, 8) \in R$, $(3, 3) \in R$, $(3, 6) \in R$, $(4, 8) \in R$, but $(2, 3) \notin R$, $(3, 8) \notin R$, $(4, 3) \notin R$, $(4, 6) \notin R$ etc.

☞ **Example 3.** If A = {1, 2, 3, 5, 7}, B = {1, 4, 6, 9} and P(x,y) : x is double of y, then R = {A, B, P(x, y)} is a relation from A into B, under which 2R4, 3R6 but $1\not R4$, $3\not R9$, etc. We can express it also as follows :

$(2, 4) \in R$, $(3, 6) \in R$, but $(1, 4) \notin R$, $(3, 9) \notin R$ etc.

❖ **Remarks :** From above examples it is obvious that

(i) It is not necessary that each element of A is related with some or the other element of B, *i.e.*, there may be such elements in A which are not related with any element of B.

(ii) An element of A way be related with one or more elements of B.

(iii) One or more elements of A may be related with one element of B.

(iv) No element of A may be related with any element of B.

(v) All the elements of A may be related with all the elements of B.

❖ **Note :** If the number of elements in A and B be m and n respectively, then the number of elements in A × B will be $m \times n$. The number of its non-empty subsets will be $2^{mn} - 1$, *i.e.*, the number of non-empty relation defined from A into B will be $2^{mn} - 1$.

1.5 Domain and Range of a Relation

❖ If R is any relation defined from set A into set B, then the set of first elements of the ordered pairs of R is called domain of relation R and the set of second elements of the ordered pairs of R is called the range of R. Hence,

Domain of R = $\{a|(a,b)\in R\}$

Range of R = $\{b|(a,b)\in R\}$

From above it is clear that the domain of R will be the subset of A and the range of R will be the subset of B.

☞ **Example 1.** If A = {2, 4, 6, 8}, B = {3, 5, 9} and a relation R from A into B is defined such that xR$y \Leftrightarrow x$ is greater than y, then,

R = {(4, 3), (6, 3), (6, 5), (8, 3), (8, 5)}

In the above relation,

Domain of R = {4, 6, 8}

Range of R = {3, 5}

☞ **Example 2.** If A = {1, 2, 3, 4, 5} and B = {2, 4, 6, 8, 10}

and let R = $\{(a, b)| a\in A, b\in B, a$ is divisor of $b\}$ is a relation from A into B, then

R = {(1, 2), (1, 4), (1, 6), (1, 8), (1, 10), (2, 2), (2, 4), (2, 6), (2, 8), (2, 10), (3, 6), (4, 4), (4, 8), (5, 10)}

Hence, domain of R = {1, 2, 3, 4, 5} = A

Range of R = {2, 4, 6, 8, 10} = B

☞ **Example 3.** A relation R is defined in Z by

R = $\{(x,y)| x, y\in Z, x^2 + y^2 \le 4\}$

domain of R = {–2, –1, 0, 1, 2}

and range of R = {–2, –1, 0, 1, 2}

☞ **Example 4.** If R = $\{(x, y) : x + 2y = 8\}$ is a relation on N, then write the range of R.

Sol. Here xR$y \Leftrightarrow x + 2y = 8$

$$\Leftrightarrow y = \frac{8-x}{2}, \; x\in N, y\in N$$

when $x = 2, y = \frac{8-2}{2} = 3\in N$

$x = 4, y = \frac{8-4}{2} = 2\in N$

$x = 6, y = \frac{8-6}{2} = 1\in N$

$x = 8, y = \frac{8-8}{2} = 0\notin N$

Hence, range of R = {1, 2, 3} **Ans.**

1.6 Inverse Relation

❖ Let R be a relation defined from set A into set B. Then the inverse relation R^{-1} of R, from set B into set A is defined as follows :

$$R^{-1} = \{(b, a) \in B \times A : (a, b)\in R\}$$

i.e. $(a, b)\in R \Leftrightarrow (b, a)\in R^{-1}$

or $aRb \Leftrightarrow bR^{-1}a$

By definition it is obvious that

domain of R^{-1} = range of R

and range of R^{-1} = domain of R

☞ **Example 1.** If A = {1, 2, 3}, and B = {0, 4} and relation R from set A into set B is defined such that

R = {(1, 0), (2, 0), (3, 0)}

Then inverse relation of R will be :

R^{-1} = {(0, 1), (0, 2), (0, 3)}

From above it is obvious that :

Domain of R^{-1} = {0} = Range of R

Range of R^{-1} = {1, 2, 3} = Domain of R

☞ **Example 2.** If relation R in N is defined by "x is less than y" then R = $\{(x,y)|x, y\in N, x < y\}$ then, its inverse relation R^{-1} "x is greater than y" is defined by $R^{-1} = \{(x,y)|x, y\in N, x > y\}$.

1.7 Types of Relations

❖ Relation are of the following types :

(i) Reflexive Relation (ii) Symmetric Relation

(iii) Transitive Relation (iv) Trivial Relation

1.8 Reflexive Relation

❖ If a relation R is defined in any set A such that under it each element of A is related with itself, then the relation R is called reflexive relation. Hence, R is a reflexive relation if and only if

$$aRa\ \forall\ a\in A$$

i.e., R is a reflexive relation

$\Leftrightarrow \quad (a, a) \in R,\ \forall\ a\in A$

❖ From the above definition it is obvious that the relation R defined in A will not be a reflexive relation if there exists at least one element a in A which is not related with itself, *i.e.*, $(a, a)\notin R$

❖ By the definition of reflexive relation R and identity relation I_A defined in any set it is clear that I_A is a subset of R, *i.e.*, it is clear that I_A is a subset of R, *i.e.*, $I_A \subseteq R$

Hence, identity relation I_A of any set A is essentially a reflexive relation in A, but it is not necessary that each reflexive relation defined in A is an identity relation.

❖ **Remark :** For a reflexive relation $(a, a)\in R$ but it does not mean that the element a is not related with any other element other than a, *i.e.*, a along with being related with itself, may be related with other elements of A also, whereas in the identity relation, a is related with a and only a. Hence, it is evident that every identity relation is a reflexive relation but every reflexive relation is not necessarily an identity relation.

☞ **Example 1.** If N is the set of natural numbers and a relative R is defined in N such that $xRy \Leftrightarrow x$ is divisor of y, $\forall x, y\in N$ then R will be a reflexive relation because every natural number is divisor of itself.

☞ **Example 2.** In the set A of straight lines lying in any plane, if a relation R be defined such that $xRy \Leftrightarrow x$ is parallel to y. Then R will be a reflexive relation because every line is parallel to itself.

☞ **Example 3.** If in the set B of triangles, a relation R is defined such that

$xRy \Leftrightarrow x$ is congruent to y, then R will be a reflexive relation because each triangle is congruent to itself.

☞ **Example 4.** In the set of sets S if a relation R is defined as follows :

ARB $\Leftrightarrow$ A is a subset of B, then R will be a reflexive relation because each set is a subset of itself.

☞ **Example 5.** In the set N of natural numbers if a relation R be defined such that $xRy \Leftrightarrow x \geq y$, then R is a reflexive relation because $x\in N \Rightarrow x = x$ but if R is defined such that $xRy \Leftrightarrow x > y$, then this relation will not be reflexive because for any elements of N, $x > x$ is not true.

☞ **Example 6.** Let A = {a, b, c, d} and

R = {(a, a), (a, d), (b, a), (b, b), (c, d), (c, c), (d, d)}

is any relation defined in A, then R is a reflexive relation because (a, a)∈R, (b, b)∈R, (c, c)∈R and (d, d)∈R but if any relation R_1 is defined in A such that R_1 = {(a, a), (a, d), (b, c), (b, d), (c, c), (c, d), (d, b)} then R_1 is not reflexive because b∈A but (b, b)$\notin R_1$. Similarly, d∈A but (d, d) $\notin R_1$

1.9 Symmetric Relation

❖ If a relation R is defined in any set A such that when a is related with b, then b is related with a by the same relation, then the relation R is called symmetric relation. Hence, R will be a symmetric relation if and only if $aRb \Rightarrow bRa$, $\forall a, b\in A$, *i.e.*, $(a, b)\in R \Rightarrow (b, a)\in R$, $\forall a, b\in A$.

From above it is clear that a relation R defined in the set A will not be symmetric if there exist two elements a and b in A such that

$$a\ R\ b \text{ but } b \not R a.$$

❖ **Note :** Inverse relation of a symmetric relation R