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Sanjiv Refresher

Mathematics

Based on the Latest CBSE Syllabus and NCERT Textbook

Class - IX

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NUMBER SYSTEMS

1.1. Introduction

- In earlier classes, we have learnt about number system, number line and how to represent various types of numbers on it.
- A number is a mathematical object used to count and measure, etc.
- Natural numbers are used for counting, *i.e.* 1, 2, 3, 4,....... the collection of natural numbers is denoted by N.

$$N = \{1, 2, 3, 4, 5, ...\}$$

- Whole numbers are all natural numbers together with zero. The collection of whole numbers is denoted by W.

$$W = \{0, 1, 2, 3, 4, ...\}$$

- Integers are all whole numbers and negative of natural numbers. The collection of integers is denoted by Z (Zahlen) or I.

$$Z \text{ or } I = \{..... -4, -3, -2, -1, 0, 1, 2, 3, 4, ...\}$$

- **Rational numbers :** Any number that can be expressed in the form $\frac{p}{q}$, where p and q are both integers and $q \neq 0$ is called a rational number.
  The word 'rational' comes from the word 'ratio'. Thus every rational number can be written as a ratio of two integers. Set of rational numbers can be expressed as Q such that :

$$Q = \left\{\frac{p}{q} : p, q \text{ are integers and } q \neq 0\right\}$$

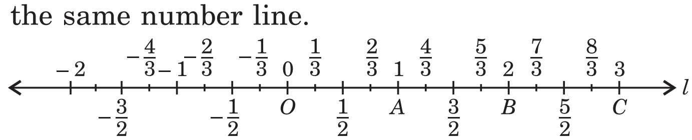
Note that every integer (positive, negative or zero) can be written in $\frac{p}{q}$ form, where $q = 1$.
For example :

$$5 = \frac{5}{1}, -7 = \frac{-7}{1}, 0 = \frac{0}{1}$$

Hence, every integer is a rational number. We know that the rational numbers do not have unique representation in the form $\frac{p}{q}$, where p and q are both integers and $q \neq 0$. For example : $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{7}{14}$, etc. In fact these are equivalent rational numbers. However, if we write a rational number in the form $\frac{p}{q}$, where p and q are both integers and $q \neq 0$ and p and q have no common factors except 1, then among the infinitely many rational numbers equivalent to $\frac{1}{2}$, we choose $\frac{1}{2}$ to represent all of them on the number line.

Thus, a rational number can be uniquely expressed as $\frac{p}{q}$; where p and q are both integers, $q \neq 0$ and p and q have no common factors except 1, *i.e.* p and q are co-prime. It is called in the lowest terms.

Representation of rational numbers : We know that natural numbers, whole numbers can be represented on a number line. We can also represent rational numbers on the same number line.



Here the length OA represents unit length. As we can see in the figure above, every integer has been represented by one and only one point on the number line l.

Next, we consider the representation of rational numbers on the line 'l'. Take one-half of the unit length and mark points on 'l' on both sides of O; these points will represent the numbers $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}$ and $\frac{-1}{2}, \frac{-2}{2}, \frac{-3}{2}$.

Similarly, take one third of the unit length and mark points on 'l' in both sides of O; these points will represent the numbers $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}$.... and $\frac{-1}{3}, \frac{-2}{3}, \frac{-3}{3}$.... and so on.

- All natural numbers, whole numbers and integers are rational numbers.
- There are infinitely many rational numbers between any two given rational numbers.

  Thus every rational number has been represented by one and only one point on the line 'l'.
- The collection of rational numbers is ordered, *i.e.* if a, b are any two rational numbers, then either $a < b$ or $a > b$ or $a = b$.
- Also, if $a < b$, then n rational numbers lying between 'a' and 'b' are $(a + d)$, $(a + 2d)$, $(a + 3d)$, ... $(a + nd)$; where, $d = \frac{b-a}{n+1}$.

For example : If we have to find 4 rational numbers between – 1 and – 2, we have $a < b$ numbers are $(a + d)$; $(a + 2d)$; $(a + 3d)$; $(a + 4d)$.

Here, $a = -2,\ b = -1$

Where, $d = \dfrac{b-a}{n+1} = \dfrac{-1-1(-2)}{4+1} = \dfrac{-1+2}{5} = \dfrac{1}{5}$

Rational numbers are

$$\left(-2+\frac{1}{5}\right);\ \left(-2+\frac{2}{5}\right);\ \left(-2+\frac{3}{5}\right);\ \left(-2+\frac{4}{5}\right) \Rightarrow \frac{-9}{5}, \frac{-8}{5}, \frac{-7}{5}, \frac{-6}{5}$$

Hence 4 rational numbers between –1 and –2 are (–1), $\frac{-9}{5}, \frac{-8}{5}, \frac{-7}{5}, \frac{-6}{5}$, (– 2).

Another method of representing rational numbers between any two rational numbers :

Let a and b be any two rational numbers, then to find a rational number between a and b. Add a and b and divide the sum by 2, *i.e.* if a, b are any two different rational numbers, then $\frac{a+b}{2}$ is a rational number and it lies between them, *i.e.* $a < \frac{a+b}{2} < b$. Continuing this process, we find that there are infinitely many rational numbers between two rational numbers.

For example : $\dfrac{\frac{1}{3}+\frac{2}{7}}{2} = \dfrac{13}{42}$ is a rational number which lies between $\frac{1}{3}$ and $\frac{2}{7}$.

- One more method of finding 'n' rational numbers between any two rational numbers :

To insert 'n' rational numbers between two rational numbers we proceed as under.

Case I : When the given rational numbers have the same denominators, multiply the numerator and denominator of each rational number by $(n + 1)$.

Case II : When the given rational numbers have different denominators.

(i) Find the LCM of denominators.

(ii) Change the given rational numbers to equivalent rational numbers with the LCM as their common denominators.

(iii) If necessary, multiply the numerator and denominator of each rational number by $(n + 1)$.

For example :

Case I : Insert 3 rational numbers between $\frac{5}{7}$ and $\frac{6}{7}$.

Multiply and divide both by $(n + 1)$, *i.e.* $3 + 1 = 4$

$$\frac{5}{7}\times\frac{4}{4} = \frac{20}{28}$$

$$\frac{6}{7}\times\frac{4}{4} = \frac{24}{28}$$

Now 3 rational numbers between $\frac{20}{28}$ and $\frac{24}{28}$ are $\frac{21}{28}, \frac{22}{28}, \frac{23}{28}$.

Case II : Insert 3 rational numbers between $\frac{2}{9}$ and $\frac{3}{7}$.
LCM of 7 and 9 = 63.

$$\frac{3}{7} \times \frac{9}{9} = \frac{27}{63}$$

$$\frac{2}{9} \times \frac{7}{7} = \frac{14}{63}$$

$\therefore$ 3 rational numbers between $\frac{14}{63}$ and $\frac{27}{63}$ are $\frac{15}{63}, \frac{16}{63}, \frac{17}{63}$.

Example

Example 1. Find nine rational numbers between 0 and 0.1.

Solution : Here, 0.1 > 0.
So, let $a = 0$, $b = 0.1$ and $n = 9$

Now, $$d = \frac{b-a}{n+1} = \frac{0.1-0}{9+1} = \frac{0.1}{10} = 0.01.$$

So, the nine rational numbers between 0 and 0.1 are :
$(x + d), (x + 2d), (x + 3d), (x + 4d), (x + 5d), (x + 6d), (x + 7d), (x + 8d)$ and $(x + 9d)$,
i.e. $(0 + 0.01), (0 + 2 \times 0.01), (0 + 3 \times 0.01), (0 + 4 \times 0.01), (0 + 5 \times 0.01),$
$(0 + 6 \times 0.01), (0 + 7 \times 0.01), (0 + 8 \times 0.01)$ and $(0 + 9 \times 0.01)$
= 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08 and 0.09

$$= \frac{1}{100}, \frac{2}{100}, \frac{3}{100}, \frac{4}{100}, \frac{5}{100}, \frac{6}{100}, \frac{7}{100}, \frac{8}{100} \text{ and } \frac{9}{100}$$

$$= \frac{1}{100}, \frac{1}{50}, \frac{3}{100}, \frac{1}{25}, \frac{1}{20}, \frac{3}{50}, \frac{7}{100}, \frac{2}{25} \text{ and } \frac{9}{100}$$

NCERT Exercise 1.1

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Solution : Yes, 0 is a rational number.
0 can be written in any of the following forms :

$\frac{0}{1}, \frac{0}{-2}, \frac{0}{3}, \frac{0}{-4}$ and so on.

Thus, 0 can be written as $\frac{p}{q}$, where $p = 0$ and q is any non-zero integer.

Hence, 0 is a rational number.

2. Find six rational numbers between 3 and 4.

Solution : We know that between two rational numbers a and b, such that $a < b$.
There is always a rational number $\frac{a+b}{2}$, where $a < \frac{a+b}{2} < b$.

A rational number between 3 and 4 is $\frac{1}{2}(3 + 4)$, *i.e.* $\frac{7}{2}$.

$\therefore \quad 3 < \frac{7}{2} < 4.$

Now, a rational number between 3 and $\frac{7}{2}$ is :

$$\frac{1}{2}\left(3+\frac{7}{2}\right) = \frac{1}{2}\times\frac{6+7}{2} = \frac{13}{4}.$$

A rational number between $\frac{7}{2}$ and 4 is :

$$\frac{1}{2}\left(\frac{7}{2}+4\right) = \frac{1}{2}\times\frac{7+8}{2} = \frac{15}{4}.$$

$\therefore \quad 3 < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} < 4$

Further, a rational number between 3 and $\frac{13}{4}$ is :

$$\frac{1}{2}\left(3+\frac{13}{4}\right) = \frac{1}{2}\times\frac{12+13}{4} = \frac{25}{8}.$$

A rational number between $\frac{15}{4}$ and 4 is :

$$\frac{1}{2}\left(\frac{15}{4}+4\right) = \frac{1}{2}\times\frac{15+16}{4} = \frac{31}{8}.$$

A rational number between $\frac{31}{8}$ and 4 is :

$$\frac{1}{2}\left(\frac{31}{8}+4\right) = \frac{1}{2}\times\frac{31+32}{8} = \frac{63}{16}.$$

$\therefore \; 3 < \frac{25}{8} < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} < \frac{31}{8} < \frac{63}{16} < 4$

Hence, six rational numbers between 3 and 4 are :

$\frac{25}{8}, \frac{13}{4}, \frac{7}{2}, \frac{15}{4}, \frac{31}{8}$ and $\frac{63}{16}$. **Ans.**

Aliter : There can be infinitely many rational numbers between the numbers 3 and 4, one way is to multiply and divide by 6 + 1 = 7, to get an equivalent fraction like :

$$3 \times \frac{7}{7} = \frac{21}{7},$$

$$4 \times \frac{7}{7} = \frac{28}{7}.$$

We know that 21 < 22 < 23 < 24 < 25 < 26 < 27 < 28

$\Rightarrow \quad \frac{21}{7} < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < \frac{28}{7}$

Hence, six rational numbers between 3 and 4 are :

$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}$ and $\frac{27}{7}$. **Ans.**

3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Solution : Since, we want 5 rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$, so we write :