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Sanjiv Refresher

Mathematics

For Basic and Standard Syllabus

Class - X

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Real Numbers

1.1. Introduction

- **Algorithm :** Algorithm is a series of well defined steps which gives a procedure for solving a type of problem.
- **Lemma :** A lemma is a proven statement used for proving another statement.
- **Divisibility of Integers :** Any positive integer 'a' can be divided by another positive integer 'b' in such a way that it leaves remainder 'r' that is smaller than 'b'.
- **Product of Primes :** Every composite number can be expressed as a product of its primes.

1.2. The Fundamental Theorem of Arithmetic

"Every composite number can be expressed as a product of primes and this factorisation is unique apart from the order in which the prime factors occur."

For any two positive integers a and b, HCF $(a, b) \times$ LCM $(a, b) = a \times b$.

❖ **Important Points :**

(i) Euclid's division algorithm is not only useful in computing the HCF of large numbers but also important for the reason that this is one of more algorithms that were first of all used as a program in a computer.

(ii) Euclid's division lemma and Euclid's division algorithm are so closely interlinked that people often call Euclid's division lemma as Euclid's division algorithm.

(iii) Euclid's division lemma/algorithm is stated only for positive integers. However it can be applied for all integers (except zero, *i.e.*, $b \neq 0$).

EXAMPLES

Example 1. Using Fundamental Theorem of Arithmetic, find the LCM and HCF of 816 and 170. ***[CBSE 2010]***

Solution : According to Fundamental Theorem of Arithmetic, every composite number can be expressed (or factorised) as a product of primes, which is unique, apart from the order in which the prime factors occur.

The prime factors of 816 and 170 gives us

$$816 = 2 \times 2 \times 2 \times 2 \times 3 \times 17 = 2^4 \times 3 \times 17$$

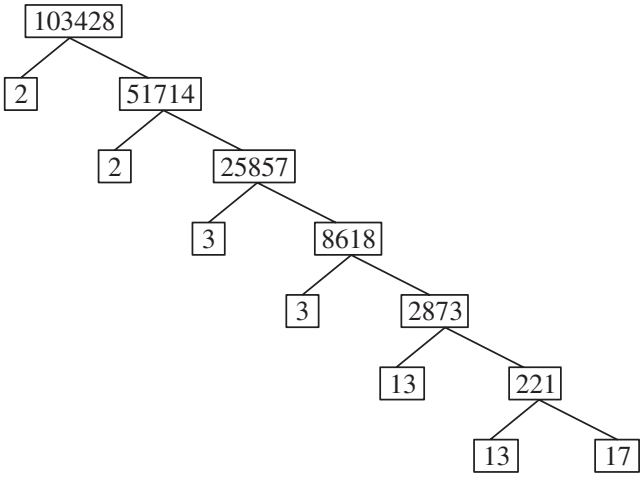
and $$170 = 2 \times 5 \times 17$$

$$\text{HCF}\ (816, 170) = 2 \times 17 = 34$$

and $$\text{LCM}\ (816, 170) = 2^4 \times 3 \times 5 \times 17 = 16 \times 15 \times 17 = 4080$$ **Ans.**

Example 2. Write the prime factorization of 103428 by using the factorisation tree.

Solution :



Therefore, the required factorization can be expressed as :

$$103428 = 2 \times 2 \times 3 \times 3 \times 13 \times 13 \times 17$$

$$= 2^2 \times 3^2 \times 13^2 \times 17^1$$ **Ans.**

Example 3. The HCF of two numbers a and b is 5 and their LCM is 200. Find the product ab. ***[CBSE 2019]***

Solution : HCF of a and b = 5

LCM of a and b = 200

We know that

product of two numbers = HCF × LCM

$$a \times b = 5 \times 200$$

$$\Rightarrow \quad ab = 1{,}000$$ **Ans.**

Example 4. Find the least number which when divided by 12, 16 and 24 leaves the remainder 7 in each case. ***[CBSE 2023]***

Solution : The least number which is divisible by 12, 16 and 24 is the LCM of 12, 16 and 24.

LCM of 12, 16 and 24 = 48

So the least number which when divided by 12, 16 and 24 leaves the remainder 7 in each case = 48 + 7

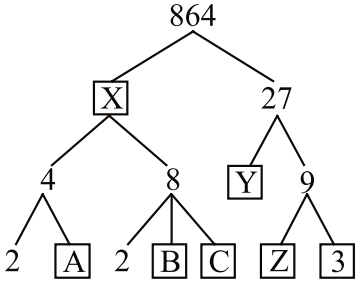
= 55 **Ans.**

Example 5. A circular field has a circumference of 360 km. Two cyclists Seema and Jyoti start together and can cycle at speeds of 12 km/hr and 15 km/hr respectively, round the circular field. After how many hours will they meet again at the starting point?

Solution : Speed of Seema = 12 km/hr

CASE STUDY BASED QUESTIONS

1. Observe the factor tree below and answer the following questions :



(i) What will be the value of X?
(a) 41 (b) 32 (c) 64 (d) 9

(ii) What will be the value of Y?
(a) 4 (b) 9 (c) 3 (d) 16

(iii) Is Y a prime number?
(a) Yes (b) No (c) Can't say

(iv) What will be the value of Z?
(a) 1 (b) 3 (c) 4 (d) 9

(v) A, B, C, Z are all ..... numbers.
(a) Composite Nos. (b) Prime Nos.
(c) Irrational Nos. (d) None of these

Ans. (i) (b) 32 (ii) (c) 3 (iii) (a) Yes (iv) (b) 3
(v) (b) Prime Nos.

2. Anu on the birthday of her mother, wanted to distribute 96 apples and 112 oranges to poor children in an orphanage. She packed all these fruits in boxes in such a way that each box contains fruits of the same kind, and also every box contains an equal number of fruits.
On the basis of the above information answer the following questions :

(i) What is the number of fruits in each box?
(a) 8 (b) 12 (c) 13 (d) 16

(ii) What is the minimum number of boxes in which all the fruits were packed?
(a) 8 (b) 12 (c) 13 (d) 16

(iii) Which concept will you use in determining answer to (i) above?
(a) Prime number (b) Factorization
(c) HCF (d) LCM

(iv) If each box of fruits costs ₹ 120, what is the total cost incurred to Anu?
(a) ₹1560 (b) ₹1920 (c) ₹2020 (d) ₹2140

(v) If each box of fruits is shared equally among four children, how many fruits does each child got?
(a) 4 (b) 8 (c) 24 (d) 52

Ans. (i) (d) 16 [$\because$ HCF of 96 and 112 is 16]
(ii) (c) 13 [96 ÷ 16 = 6; 112 ÷ 16 = 7; 6 + 7 = 13]
(iii) (c) HCF
(iv) (a) ₹1560 [$\because$ 13 × 120 = 1560]
(v) (a) 4 [$\because$ 16 ÷ 4 = 4]

3. Indian Army is the third biggest military contingent in the world next to USA and China. However, there are many firsts that make Indian army stand out in the world, making us all Indians very proud. Knowing them, will help you celebrate Republic Day with greater vigour and gratitude.

On 71th Republic Day parade in Delhi Captain RS Meel is planning for parade of following two group :

(a) First group of Army contingent of 624 members behind an army band of 32 members.

(b) Second group of CRPF troops with 468 soldiers behind the 228 members of bikers. These two groups are to march in the same number of columns. This sequence of soldiers is followed by different states Jhanki which are showing the culture of the respective states.

(i) What is the maximum number of columns in which the army troop can march?

(a) 8 (b) 16 (c) 4 (d) 32

(ii) What is the maximum number of columns in which the CRPF troop can march?

(a) 4 (b) 8 (c) 12 (d) 16

(iii) What is the maximum number of columns in which total army troop and CRPF troop together can march past?

(a) 2 (b) 4 (c) 6 (d) 8

(iv) What should be subtracted with the number of CRPF soldiers and the number of bikers so that their maximum number of column is equal to the maximum number of column of army troop?

(a) 4 soldiers and 4 bikers (b) 4 soldiers and 2 bikers
(c) 2 soldiers and 4 bikers (d) 2 soldiers and 2 bikers

(v) What should be added with the number of CRPF soldiers and the number of bikers so that their maximum number of column is equal to the maximum number of column of army troop?

(a) 4 soldiers and 4 bikers (b) 12 soldiers and 12 bikers
(c) 6 soldiers and 6 bikers (d) 12 soldiers and 6 bikers

Ans. (i) We will find the HCF (624, 32) = 16
Thus (b) is correct option.

(ii) We will find the HCF (228, 468) = 12
Thus (c) is correct option.

(iii) According to the question, we have to find out
HCF (625, 32, 228, 468) = 4
Alternatively we can find,
HCF (16, 12) = 4
Thus (b) is correct option.

(iv) Maximum number of column of army troop is 16. But 228 and 468 are not divisible by 16. If we subtract 4 from 228 and 468, both (224 and 464) are divisible by 16.
Thus (a) is correct option.

(v) Maximum number of column of army troop is 16. But 228 and 468 are not divisible by 16. If we add 12 to 228 and 468, both (240 and 480) are divisible by 16.
Thus (b) is correct option.

4. The department of Computer Science and Technology is conducting an International Seminar. In the seminar, the number of participants in Mathematics, Science and Computer Science are 60, 84 and 108 respectively. The coordinator has made the arrangement such that in each room, the same number of participants are to be seated and all of them being in the same subject. Also, they allotted the separate room for all the official other than participants.

On the basis of given observation, answer the following questions :

(i) Find the total number of participants :

(a) 60 (b) 84 (c) 108 (d) None of these

(ii) Find the LCM of 60, 84 and 108 :

(a) 12 (b) 504 (c) 544320 (d) 3780

(iii) Find the HCF of 60, 84 and 108 :

(a) 12 (b) 60 (c) 84 (d) 108

(iv) Find the minimum number of rooms required, if in each room, the same number of participants are to be seated and all of them being in the same subject :

(a) 12 (b) 20 (c) 21 (d) None of these

(v) Based on the above (iv) condition, find the minimum number of rooms required for all the participants and officials :

(a) 12 (b) 20 (c) 21 (d) None of these

Ans. (i) (d) None of these

Sol. Total number of participants

$= 60 + 84 + 108 = 252$

(ii) (d) 3780

Sol. $60 = 2^2 \times 3 \times 5$

$84 = 2^2 \times 3 \times 7$

$108 = 2^2 \times 3^3$

$LCM = 2^2 \times 3^3 \times 5 \times 7$

$= 3780$

(iii) (a) 12

Sol. $60 = 2^2 \times 3 \times 5$

$84 = 2^2 \times 3 \times 7$

$108 = 2^2 \times 3^3$

(iv) (c) 21

Sol. Minimum number of rooms required for all the participants = 252/12 = 21.

(v) (d) None of these

Sol. Minimum number of rooms required for all = 21 + 1 = 22.

5. Khushi wants to organize her birthday party. Being health conscious, she decided to serve only fruits in her birthday party. She bought 36 apples and 60 bananas and decided to distribute fruits equally among all.

Based on the above information, answer the following questions :

(i) How many guests Khushi can intive at the most?

(ii) How many apples and bananas will each guest get?

(iii) If Khushi decides to add 42 mangoes, how many guests Khushi can intvite at the most? ***[CBSE 2023]***

Ans. Khushi decided to distribute fruits equally among all. She had 36 apples and 60 bananas.

(i) The number of guests, Khushi can invite at the most for this we find HCF of 36 and 60.

HCF = 12

Hence, the most number of guests are 12.

(ii) Number of apples will each guest get = $\frac{36}{12}$ = 3 apples.

Number of bananas will each guest get = $\frac{60}{12}$ = 5 bananas.

(iii) If she add 42 mangoes,

Number of guests Khushi can invite at the most = HCF of 36, 60 and 42

HCF = 6

Hence, the most number of guests are 6.

PRACTICE EXERCISE

I. Very Short Answer Type Questions :

1. If the prime factorisation of a natural number 'N' is $2^4 \times 3^4 \times 5^3 \times 7$, write the number of consecutive zeroes in N.
2. If a and b are two positive integers such that $a = 14b$, find the HCF of a and b.
3. Find the exponent of 2 in the prime factorisation of 288. ***[CBSE 2021]***
4. If $x = 2^3 \times 3 \times 5^2$ and $y = 2^2 \times 3^3$, then find HCF (x, y). ***[CBSE 2010]***
5. What is the HCF of smallest prime number and the smallest composite number? ***[CBSE 2018]***
6. The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, find the other. ***[CBSE 2020]***

II. Short Answer (I) Type Questions :

7. The HCF of two numbers is 23 and their LCM is 1449. If one of the numbers is 161, find the other.
8. Find the least positive integer divisible by first five natural numbers. ***[CBSE 2014]***
9. Given that $\sqrt{2}$ is irrational, prove that $(5 + 3\sqrt{2})$ is an irrational number. ***[CBSE 2018]***
10. Find the LCM of 2.5, 0.5 and 0.175.
11. Show that $5 + 2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number. ***[CBSE 2020]***
12. Check whether 12^n can end with the digit 0 for any natural number n. ***[CBSE 2020]***

III. Short Answer (II) Type Questions :

13. If the HCF of 657 and 963 is expressible in the form of $657x + 963 (-15)$, then find x. ***[CBSE 2010]***
14. Find the greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively. ***[CBSE 2010, 08]***