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Class VIII

Based on Latest NCERT Textbook

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Syllabus

No.	Chapter/Unit	Objectives
1.	Rational Numbers	The students will be able to clear all their confusion and learn about rational numbers and their use in everyday life.
2.	Linear Equations in One Variable	Students can get into the heart of what algebra is all about solving equations and will learn linear equations in one variable.
3.	Understanding Quadrilaterals	Learn in depth about quadrilaterals of regular and irregular shapes.
4.	Data Handling	Representation of data with the help of graphs and tables.
5.	Squares and Square Roots	Learn about different methods of calculating the square root of numbers.
6.	Cubes and Cube Roots	Learn about the different steps and tricks for finding cube roots which will make learning easy to students.
7.	Comparing Quantities	Use of percentage, ratio, compound interest and discount.
8.	Algebraic Expressions and Identities	Some types of algebraic expressions like the monomials, binomials, trinomials, and polynomials and apply some operations on these expressions.
9.	Mensuration	Find the area and perimeters of various plane figures.
10.	Exponents and Powers	Learn about different types of exponents right from the fundamental way of adding and subtracting the numbers with the same power or the same numbers having different powers.
11.	Direct and Inverse Proportions	Help the students learn more about the direct and indirect proportion and conditions that are required for the quantities to be in direct or indirect proportion.
12.	Factorisation	Helps to build the basics and in-depth understanding of the basic mathematical concepts of factors.
13.	Introduction to Graphs	Basic learning of making graphs.

Contents

Rational Numbers

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CHAPTER

1.1. Introduction

In previous classes, we began our study of numbers with counting numbers or natural numbers, i.e. 1, 2, 3, 4...... By including 0 to natural numbers, we got whole numbers, i.e. 0, 1, 2, 3, 4.......

The negative of natural numbers were put together with whole numbers to get integers, i.e. .... – 4, – 3, – 2, – 1, 0, 1, 2, 3, ...

In class VII, the concept of rational numbers was introduced and addition, subtraction, multiplication and division on the rational numbers were defined. In this chapter, we will learn about various properties of these operations on the rational numbers.

Rational Number

A number of the form $\frac{p}{q}$ or a number which can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number.

Example : Each of the numbers $\frac{5}{8}, \frac{-3}{14}, 15$ and 0 are rational numbers.

Positive rational number : A rational number is said to be positive if its numerator and denominator are of same signs.

Example : $\frac{3}{14}$ and $\frac{-7}{-15}$ are positive rational numbers.

Negative rational number : A rational number is said to be negative if its numerator or denominator are of opposite signs.

Example : $\frac{-2}{9}$ and $\frac{5}{-7}$ are negative rational numbers.

Equivalent rational numbers : If $\frac{p}{q}$ is a rational number and m is a non-zero integer, then

$$\frac{p}{q} = \frac{p \times m}{q \times m},$$

$\frac{p \times m}{q \times m}$ is a rational number equivalent to $\frac{p}{q}$.

Example : $\frac{-3}{5} = \frac{(-3) \times 2}{5 \times 2} = \frac{(-3) \times 3}{5 \times 3} = \frac{(-3) \times 4}{5 \times 4} = ...$

$\therefore \frac{-3}{5}, \frac{-6}{10}, \frac{-9}{15}, \frac{-12}{20}$, etc. are equivalent rational numbers.

Standard form of a rational number : A rational number $\frac{a}{b}$ is said to be in standard form if a and b are integers having no common divisor other than 1 and b is positive.

Example : Express $\frac{21}{-56}$ in standard form.

Solution : $$\frac{21}{-56} = \frac{21 \times -1}{-56 \times -1} = \frac{-21}{56}$$

The greatest common divisor of 21 and 56 is 7

$$\therefore \quad \frac{-21}{56} = \frac{(-21) \div 7}{56 \div 7} = \frac{-3}{8}$$

Hence, $\frac{-3}{8}$ is the standard form of number $\frac{21}{-56}$.

Comparison of Rational Numbers

(i) Every positive rational number is greater than 0.

(ii) Every negative rational number is less than 0.

Example : Which of the numbers $\frac{3}{-4}$ or $\frac{-5}{6}$ is greater?

Solution : First we write each of the given numbers with positive denominator.

$$\text{One number} = \frac{3}{-4} = \frac{3 \times (-1)}{-4 \times (-1)} = \frac{-3}{4}$$

$$\text{The other number} = \frac{-5}{6}$$

LCM of 4 and 6 = 12

$$\therefore \frac{-3}{4} = \frac{(-3) \times 3}{4 \times 3} = \frac{-9}{12} \text{ and } \frac{-5}{6} = \frac{(-5) \times 2}{6 \times 2} = \frac{-10}{12}$$

Since, $-9 > -10 \Rightarrow \frac{-9}{12} > \frac{-10}{12}$

Hence, $\frac{-3}{4} > \frac{-5}{6}$ or $\frac{3}{-4} > \frac{-5}{6}$,

i.e. $\frac{3}{-4}$ is greater.

Example : Arrange the numbers $\frac{-3}{5}, \frac{7}{-10}, \frac{-5}{8}$ in ascending order.

Solution : First we write each of the given numbers with positive denominator.

We have, $\frac{7}{-10} = \frac{7 \times (-1)}{(-10) \times (-1)} = \frac{-7}{10}$

Thus, the given numbers are $\frac{-3}{5}, \frac{-7}{10}$ and $\frac{-5}{8}$.

LCM of 5, 10 and 8 is 40.

Now, $\frac{-3}{5} = \frac{(-3) \times 8}{5 \times 8} = \frac{-24}{40}$; $\frac{-7}{10} = \frac{(-7) \times 4}{10 \times 4} = \frac{-28}{40}$

and, $\frac{-5}{8} = \frac{(-5) \times 5}{8 \times 5} = \frac{-25}{40}$

Clearly, $\frac{-28}{40} < \frac{-25}{40} < \frac{-24}{40}$

Hence, $\frac{-7}{10} < \frac{-5}{8} < \frac{-3}{5}$, i.e. $\frac{7}{-10} < \frac{-5}{8} < \frac{-3}{5}$.

1.2. Properties of Rational Numbers

(i) Addition of Rational Numbers

If two rational numbers are to be added, we should convert each of them into a rational number with positive denominator.

Case I : When denominators are same :

In this case, $\left(\frac{a}{b} + \frac{c}{b}\right) = \frac{(a+c)}{b}$

Example : Find the sum (i) $\frac{-4}{7} + \frac{5}{7}$, (ii) $\frac{8}{-13} + \frac{5}{13}$.

Solution :

(i) $\frac{-4}{7} + \frac{5}{7} = \frac{(-4)+5}{7} = \frac{1}{7}$

(ii) $\frac{8}{-13} = \frac{8 \times (-1)}{(-13) \times (-1)} = \frac{-8}{13}$

Now, $\frac{-8}{13} + \frac{5}{13} = \frac{(-8)+5}{13} = \frac{-3}{13}$

Case II : When denominators are unequal :

In this case first we will make the denominators same by expressing these numbers with the LCM of the denominators as a common denominator. Now, we add these numbers as shown in case I.

Example : Find the sum $\frac{-9}{16} + \frac{5}{12}$.

Solution : LCM of 16 and 12 = 48

$$\therefore \quad \frac{-9}{16} + \frac{5}{12} = \frac{3 \times (-9) + 4 \times 5}{48}$$

$$= \frac{(-27)+20}{48} = \frac{-7}{48}$$

Properties of Addition of Rational Numbers

Property 1 : Closure property : The sum of two rational numbers is always a rational number. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\left(\frac{a}{b}+\frac{c}{d}\right)$ is also a rational number.

Example : Consider $\frac{-4}{3}$ and $\frac{3}{5}$ are rational numbers, then $\left(\frac{-4}{3}+\frac{3}{5}\right)=\frac{-20+9}{15}=\frac{-11}{15}$ is also a rational number.

Property 2 : Commutative property : For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$

$$\left(\frac{a}{b}+\frac{c}{d}\right)=\left(\frac{c}{d}+\frac{a}{b}\right).$$

Example : For $\frac{a}{b}=\frac{3}{7};\ \frac{c}{d}=\frac{-5}{8}$

$$\left(\frac{a}{b}+\frac{c}{d}\right)=\left\{\frac{3}{7}+\frac{(-5)}{8}\right\}=\left\{\frac{24+(-35)}{56}\right\}=\frac{-11}{56}$$

Now,
$$\left(\frac{c}{d}+\frac{a}{b}\right)=\left(\frac{-5}{8}+\frac{3}{7}\right)$$
$$=\frac{-35+24}{56}=\frac{-11}{56}$$

Thus,
$$\left(\frac{a}{b}+\frac{c}{d}\right)=\left(\frac{c}{d}+\frac{a}{b}\right).$$

Property 3 : Associative property : While adding three rational numbers, they can be grouped in any order. Thus, for any three rational numbers $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$,

we have, $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right).$

Example : For $\frac{a}{b}=\frac{-2}{3}, \frac{c}{d}=\frac{5}{6}$ and $\frac{e}{f}=\frac{1}{2}$.

we have
$$\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\left\{\left(\frac{-2}{3}+\frac{5}{6}\right)+\frac{1}{2}\right\}$$
$$=\left\{\left(\frac{-4+5}{6}\right)+\frac{1}{2}\right\}=\left\{\frac{1}{6}+\frac{1}{2}\right\}$$
$$=\left(\frac{1+3}{6}\right)=\frac{4}{6}=\frac{2}{3}$$
$$\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)=\left\{\frac{-2}{3}+\left(\frac{5}{6}+\frac{1}{2}\right)\right\}$$
$$=\left\{\frac{-2}{3}+\left(\frac{5+3}{6}\right)\right\}=\left\{\frac{-2}{3}+\frac{8}{6}\right\}$$
$$=\left(\frac{-4+8}{6}\right)=\frac{4}{6}=\frac{2}{3}$$

Thus, $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right).$

Property 4 : Existence of additive identity : If 0 is added to any rational number, then always of rational number itself.

$\frac{a}{b} + 0 = \frac{a}{b} = 0 + \frac{a}{b}, \frac{a}{b}$

Hence, 0 is called the additive identity for rational numbers.

Example : $\left(\frac{3}{7} + 0\right) = \frac{3}{7} = \left(0 + \frac{3}{7}\right)$

(ii) Subtraction of Rational Numbers

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then subtracting $\frac{c}{d}$ means adding additive inverse of $\frac{c}{d}$ to $\frac{a}{b}$ correct it.

Thus, $\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \left(\frac{-c}{d}\right)$

Example : Subtract $\frac{3}{2}$ from $\frac{5}{7}$.

Solution : $\frac{5}{7} + \left(\frac{-3}{2}\right) = \frac{10 + (-21)}{14} = \frac{10 - 21}{14} = \frac{-11}{14}$

Properties of Subtraction of Rational Numbers

Property 1 : Closure property : If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers then $\frac{a}{b} - \frac{c}{d}$ is also a rational number.

Example : If $\frac{1}{2}$ and $\frac{-3}{7}$ are rational numbers, then

$$\left\{\frac{1}{2} - \left(\frac{-3}{7}\right)\right\} = \left\{\frac{1}{2} + \frac{3}{7}\right\}$$

$$= \left\{\frac{7+6}{14}\right\} = \frac{13}{14}$$ is also a rational number.

Property 2 : Commutative property : For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$,

$$\left(\frac{a}{b} - \frac{c}{d}\right) \neq \left(\frac{c}{d} - \frac{a}{b}\right)$$

Hence, rational numbers are not commutative under subtraction.

Example : For $\frac{a}{b} = \frac{2}{3}, \frac{c}{d} = \frac{1}{6}$,

$$\frac{a}{b} - \frac{c}{d} = \frac{2}{3} + \frac{-1}{6} = \frac{4-1}{6} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{c}{d} - \frac{a}{b} = \frac{1}{6} + \frac{-2}{3} = \frac{1-4}{6} = \frac{-3}{6} = \frac{-1}{2}$$

Thus, $$\left(\frac{a}{b} - \frac{c}{d}\right) \neq \left(\frac{c}{d} - \frac{a}{b}\right)$$

Property 3 : Associative property : For any three rational numbers $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$,

$$\left(\frac{a}{b} - \frac{c}{d}\right) - \frac{e}{f} \neq \frac{a}{b} - \left(\frac{c}{d} - \frac{e}{f}\right).$$

Hence, rational numbers are not associative under subtraction.

(iii) Multiplication of Rational Numbers

For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we have

$$\left(\frac{a}{b}\times\frac{c}{d}\right) = \frac{(a\times c)}{(b\times d)}$$

Example : Multiply (i) $\frac{-3}{7}\times\frac{5}{9}$, (ii) $\frac{-11}{13}\times\frac{2}{5}$.

Solution : (i) $\frac{-3}{7}\times\frac{5}{9}=\frac{(-3)\times 5}{7\times 9}=\frac{-15}{63}=\frac{-5}{21}$

(ii) $\frac{-11}{13}\times\frac{2}{5}=\frac{(-11)\times 2}{13\times 5}=\frac{-22}{65}$

Properties of Multiplication of Rational Numbers

Property 1 : Closure property : The product of two rational numbers is always a rational number.

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers then $\left(\frac{a}{b}\times\frac{c}{d}\right)$ is also a rational number.

Example : For two rational numbers $\frac{1}{2}$ and $\frac{3}{7}$.

$\frac{1}{2}\times\frac{3}{7}=\frac{1\times 3}{2\times 7}=\frac{3}{14}$ is also a rational number.

Property 2 : Commutative property : For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$

$$\left(\frac{a}{b}\times\frac{c}{d}\right) = \left(\frac{c}{d}\times\frac{a}{b}\right).$$

Example : Let us consider $\frac{a}{b}=\frac{5}{6}$ and $\frac{c}{d}=\frac{-4}{7}$, then $\frac{a}{b}\times\frac{c}{d}=\frac{5}{6}\times\frac{-4}{7}$

$$=\frac{5\times(-4)}{6\times 7}=\frac{-20}{42}=\frac{-10}{21}$$

Also, $\frac{c}{d}\times\frac{a}{b}=\frac{-4}{7}\times\frac{5}{6}=\frac{(-4)\times 5}{7\times 6}=\frac{-20}{42}=\frac{-10}{21}$

Hence, $\left(\frac{a}{b}\times\frac{c}{d}\right) = \left(\frac{c}{d}\times\frac{a}{b}\right)$

Property 3 : Associative property : For any three rational numbers $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$, we have,

$$\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}=\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right).$$

Example : Let us consider $\frac{a}{b}=\frac{-5}{2}$, $\frac{c}{d}=\frac{-7}{4}$ and $\frac{e}{f}=\frac{1}{3}$, we have

$$\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f} = \left\{\frac{(-5)}{2}\times\frac{(-7)}{4}\right\}\times\frac{1}{3} = \left(\frac{35}{8}\times\frac{1}{3}\right) = \frac{35}{24}$$