According to the Latest Syllabus

Sanjiv®

Mathematics

Class-12 (Part-I)

For the Students of Rajasthan Board of Secondary Education

By

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2025

Price : ₹ 480/-

Sanjiv Prakashan, Jaipur

- Publisher :
  Sanjiv Prakashan
  Dhamani Market, Chaura Rasta,
  Jaipur-3
  email : sanjeevprakashanjaipur@gmail.com
  website : www.sanjivprakashan.com

- Laser Typesetting :
  Sanjiv Prakashan (DTP Department), Jaipur

- Printer :
  Punjabi Press, Jaipur
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Preface

We are extremely pleased to present this book according to latest syllabus of NCERT. The book has been written in easy and simple language so that students may assimilate the subject easily. We hope that students will get benefitted from it and teachers will appreciate our efforts. In comparison to other books available in market, this book has many such features which make it a unique book :

1. Theoretical subject-material is given in adequate and accurate description along with pictures.
2. The latest syllabus of NCERT is followed thoroughly.
3. Complete solutions of all the questions given at the end of the chapter in the textbook are given in easy language.
4. Topic wise summary is also given in each chapter for the revision of the chapter.
5. In every chapter, all types of questions that can be asked in the exam (Objective, Fill in the blanks, Very short, Short, Numerical and Long answer type questions) are given.
6. At the end of every chapter, multiple choice questions asked in various competitive exams are also given with solutions.

Valuable suggestions received from subject experts, teachers and students have also been given appropriate place in the book.

We wholeheartedly bow to the Almighty God, whose continuous inspiration and blessings have made the writing of this book possible.

We express our heartfelt gratitude to the publisher – Mr. Pradeep Mittal and Manoj Mittal of Sanjiv Prakashan, all their staff, laser type center and printer for publishing this book in an attractive format on time and making it reach the hands of the students.

Although utmost care has been taken in publishing the book, human errors are still possible, hence, valuable suggestions are always welcome to make the book more useful.

In anticipation of cooperation!

Authors

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Solved Paper

SENIOR SECONDARY EXAMINATION, 2024

MATHEMATICS

General Instructions to the Examinees :

(1) Candidate must write first his/her Roll No. on the question paper compulsorily.
(2) All the questions are compulsory.
(3) Write the answer to each question in the given answer-book only.
(4) For questions having more than one part, the answers to those parts are to be written together in continuity.
(5) If there is any error/difference/contradiction in Hindi & English versions of the question paper, the question of Hindi version should be treated valid.
(6) Write down the serial number of the question before attempting it.
(7) Q. Nos. **16** to **22** having internal choices.
(8) Solve Question number 22 on graph paper.

Section-A

1. Multiple Choice Questions :

(i) Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer in the given options. **1**
(a) R is reflexive and symmetric but not transitive.
(b) R is reflexive and transitive but not symmetric.
(c) R is symmetric and transitive but not reflexive.
(d) Ris an equivalence relation.

(ii) The principal value of $\text{cosec}^{-1}(2)$ is : **1**
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) π.

(iii) If A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ then, (2A–B) will be : **1**
(a) $\begin{bmatrix} 1 & -5 & 2 \\ 5 & 6 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 6 & 0 \\ 1 & -5 & 3 \end{bmatrix}$
(c) $\begin{bmatrix} -1 & 5 & 3 \\ 5 & 6 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 3 & 5 \\ 5 & 6 & 0 \end{bmatrix}$.

(iv) If $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$; then the value of x is : **1**
(a) 2 (b) 0 (c) 1 (d) – 1.

(v) If $2x + 8y = \sin x$, then $\frac{dy}{dx}$ is : **1**
(a) $\frac{\sin x - 2}{8}$ (b) $\frac{\cos x - 2}{8}$
(c) $\frac{\cos x + 2}{2}$ (d) $\frac{\cos x + 2}{3}$.

(vi) In which of the following intervals is $y = x^2e^{-x}$ increasing ? **1**
(a) (1, 0) (b) (2, 0)
(c) $(2, -\infty)$ (d) (0, 2).

(vii) The value of $\int \frac{\sec^2 x}{\text{cosec}^2 x} dx$: **1**
(a) $\sec x - x + c$ (b) $\sec x \tan x + c$
(c) $\tan x + x^2 + c$ (d) $\tan x - x + c$.

(viii) The area of the region bounded by the circle $x^2 + y^2 = 9$ in the first quadrant is : **1**
(a) 9π (b) $\frac{3\pi}{4}$ (c) $\frac{9\pi}{4}$ (d) 3π.

(ix) Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is : **1**
(a) 2 (b) $\frac{9}{4}$
(c) $\frac{9}{8}$ (d) $\frac{9}{2}$.

(x) The degree of the differential equation $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$ is : **1**
(a) 1 (b) 2
(c) 3 (d) 4.

(xi) If $\vec{a}$ is a non-zero vector of magnitude 'a' and λ a non-zero scalar, then $\lambda\vec{a}$ is unit vector if : **1**
(a) $\lambda = 1$ (b) $\lambda = -1$
(c) $a = |\lambda|$ (d) $a = \frac{1}{|\lambda|}$.

(xii) The direction cosine of y-axis is : **1**
(a) 0, 0, 0 (b) 1, 0, 0
(c) 0, 1, 0 (d) 0, 0, 1.

(xiii) The direction cosines of the line passing through the two points (– 2, 4, – 5) and (1, 2, 3) is : **1**
(a) $\frac{3}{\sqrt{70}}, \frac{2}{\sqrt{70}}, \frac{8}{\sqrt{70}}$ (b) $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$
(c) $\frac{2}{\sqrt{77}}, \frac{-3}{\sqrt{77}}, \frac{8}{\sqrt{77}}$ (d) $\frac{8}{\sqrt{13}}, \frac{-2}{\sqrt{13}}, \frac{3}{\sqrt{13}}$.

(xiv) If P(A) = 0.8, P(B) = 0.5 and P(B/A) = 0.4, then the value of $P(A \cap B)$ is : **1**

(a) 0.32 (b) 0.20
(c) 0.40 (d) 0.64.

(xv) Two cards are drawn at random and without replacement from a pack of 52 playing cards, then the probability that both the cards are black is : **1**

(a) $\frac{26}{52}$ (b) $\frac{52}{102}$ (c) $\frac{25}{51}$ (d) $\frac{1}{2}$.

Answers

(i) (b)	(ii) (c)	(iii) (c)	(iv) (a)	(v) (b)
(vi) (d)	(vii) (d)	(viii) (c)	(ix) (b)	(x) (a)
(xi) (d)	(xii) (c)	(xiii) (b)	(xiv) (a)	(xv) (b).

2. Fill in the blanks :

(i) $\sin^{-1} x$ is a function whose domain is........... . **1**

(ii) The value of $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$ is............ . **1**

(iii) The principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$........... . **1**

(iv) If $y = \cos\sqrt{x}$, then the value of $\frac{dy}{dx}$ will be........... . **1**

(v) The rate of change of the area of a circle with respect to its radius r at $r = 3$ cm is............. . **1**

(vi) The numbers of arbitrary constants present in the particular solution of a differential equation of third order are............. . **1**

(vii) A vector whose initial and terminal points coincide, is called........... . **1**

Answers

(i) [– 1, 1], (ii) $\frac{\pi}{3}$, (iii) $\frac{\pi}{6}$, (iv) $\frac{-\sin\sqrt{x}}{2\sqrt{x}}$, (v) 6π cm^2/cm, (vi) zero, (vii) zero vector.

3. Very Short Answer Type Questions :

(i) Find the value of determinant $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$. **1**

Sol. $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \cos\theta \times \cos\theta - \sin\theta\,(-\sin\theta)$

$= \cos^2\theta + \sin^2\theta$

$= 1.$

(ii) Find equation of line joining (1, 2) and (3, 6) using determinants. **1**

Sol. Let a point P(x, y) lies on the line segment joining the ponts A (1, 2) and B (3, 6). The area of ΔABP will be zero.

$$\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

or
$$\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$1\,(6 - y) - 2(3 - x) + 1(3y - 6x) = 0$

$\Rightarrow \quad 6 - y - 6 + 2x + 3y - 6x = 0$

$-4x + 2y = 0$

or $\quad y = 2x.$

(iii) The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm. **1**

Sol. Let the area of circle be A and radius r.
According to question,

$$\frac{dr}{dt} = 3 \text{ cm/s}$$
$$A = \pi r^2$$
$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

Putting $r = 10$ cm, $\frac{dr}{dt} = 3$ cm/s

$$\frac{dA}{dt} = \frac{2\pi \times 10 \times 3}{60\pi \text{ cm}^2/\text{s}}$$ **Ans.**

(iv) Prove that the logarithmic function is increasing on $(0, \infty)$. **1**

Sol. Let $f(x) = \log_e x, x > 0$

$\therefore \quad f'(x) = \frac{1}{x} = +ve$, for $x > 0$

$\Rightarrow$ For $x \leftarrow (0, \infty), f'(x) > 0$

Therefore, the given logarithmic function is increasing in $(0, \infty)$.

(v) Evaluate $\int(2x - 3\cos x + e^x)\,dx$. **1**

Sol. Let $I = \int(2x - 3\cos x + e^x)\,dx$

$= \frac{2.x^2}{2} - 3(\sin x) + e^x + c$

$= x^2 - 3\sin x + e^x + c$ **Ans.**

(vi) Evaluate $\int \frac{\sin x}{1+\cos x}dx$. **1**

Sol. Let $I = \int \frac{\sin x}{1 + \cos x}dx$

Here, $1 + \cos x = t$, $\therefore -\sin x\,dx = dt$

$\therefore \quad I = -\int\frac{1}{t}dt = -\log|t| + c$

$= -\log|1 + \cos x| + c$

$= \log\left|\frac{1}{1+\cos x}\right| + c$ **Ans.**

(vii) Verify that the function $y = e^x + 1$ is a solution of the differential equation $y'' - y' = 0$. **1**

Sol. Given function

$$y = e^x + 1$$

Differentiating with respect to x,

$$y' = e^x + 0 = e^x$$

or $y' = y - 1$ [By putting $y = e^x$]

Again differentiating with respect to x

$$y'' = y' - 0 = y'$$

$\Rightarrow$ $y'' = y'$ **Hence proved.**

(viii) Find the position vector of the mid-point of the vector joining the points P(2, 3, 4) and Q(4, 1, – 2). **1**

Sol. We know that the position vector of joining two points $P(\vec{a})$ and $Q(\vec{b})$ is $\frac{\vec{a}+\vec{b}}{2}$. Hence according to question,

$$\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k},\ \vec{b} = 4\hat{i} + \hat{j} - 2\hat{k}$$

$$= \frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) + (4\hat{i} + \hat{j} - 2\hat{k})}{2}$$

$$= \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2} = 3\hat{i} + 2\hat{j} + \hat{k}$$ **Ans.**

(ix) Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. **1**

Sol. Projection of $\vec{a}$ or $\vec{b}$ = $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$= \frac{(2\hat{i} + 3\hat{j} + 2\hat{k})\cdot(\hat{i} + 2\hat{j} + \hat{k})}{\sqrt{(1)^2 + (2)^2 + (1)^2}}$$

$$= \frac{2\cdot1 + 3\cdot2 + 2\cdot1}{\sqrt{6}} = \frac{2+6+2}{\sqrt{6}} = \frac{10}{\sqrt{6}} = \frac{5}{3}\sqrt{6}$$

Ans.

(x) Evaluate the product $(3\vec{a} - 5\vec{b})\cdot(2\vec{a} + 7\vec{b})$. **1**

Sol. According to question,

$$(3\vec{a} - 5\vec{b})\cdot(2\vec{a} + 7\vec{b})$$

$$\Rightarrow (3\vec{a})\cdot(2\vec{b}) + (3\vec{a})\cdot(7\vec{b}) + (-5\vec{b})\cdot(2\vec{a}) + (-5\vec{b})\cdot(7\vec{b})$$

$$= 6\vec{a}\cdot\vec{a} + 21\vec{a}\cdot\vec{b} - 10\vec{b}\cdot\vec{a} - 35\vec{b}\cdot\vec{b}$$

$$= 6|\vec{a}|^2 + 21(\vec{a}\cdot\vec{b}) - 10(\vec{b}\cdot\vec{a}) - 35|\vec{b}|^2$$

$$\because \vec{a}\cdot\vec{a} = |\vec{a}|^2,\ \vec{b}\cdot\vec{b} = |\vec{b}|^2,\ \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$$

So, $(3\vec{a} - 5\vec{b})\cdot(2\vec{a} + 7\vec{b})$

$$= 6|\vec{a}|^2 + 21(\vec{a}\cdot\vec{b}) - 10\vec{a}\cdot\vec{b} - 35|\vec{b}|^2$$

$$= 6|\vec{a}|^2 + 11\vec{a}\cdot\vec{b} - 35|\vec{b}|^2$$ **Ans.**

Section–B

Short Answer Type Questions :

4. Prove that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive. **2**

Sol. Given set A = {1, 2, 3}

Relation R = {(1, 2), (2, 1)}

R is not reflexive because (1, 1), (2, 2), (3, 3) $\notin$ R

R is symmetric because (1, 2) $\in$ R $\Rightarrow$ (2, 1) $\in$ R

R is not transitive because (1, 2), (2, 1) $\in$ R but (1, 1) $\notin$ R

Hence R is symmetric but R is not reflexive and transtive.

5. Simplify $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$. **2**

Sol. According to question,

$$\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \cos\theta\sin\theta \\ -\cos\theta\sin\theta + \cos\theta\sin\theta & \cos^2\theta + \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ **Ans.**

6. Show that $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$. **2**

Sol. To prove

$$\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$\text{LHS} = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5\times2 + (-1)\times3 & 5\times1 + (-1)\times4 \\ 6\times2 + 7\times3 & 6\times1 + 7\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix} \quad ...(1)$$

$$\text{RHS} = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times5+1\times6 & 2\times(-1)+1\times7 \\ 3\times5+4\times6 & 3\times(-1)+4\times7 \end{bmatrix}$$

$$= \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \quad ...(2)$$

From equation (1) and (2)

$$\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$ **Ans.**

7. Find the adjoint of matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. **2**

Sol. Given matrix

$$A = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

$A_{11} = (-1)^{1+1} M_{11} = 4$
$A_{12} = (-1)^{1+2} M_{12} = -3$
$A_{21} = (-1)^{2+1} M_{21} = -2$
$A_{22} = (-1)^{2+2} M_{22} = 1$

$$\text{Matrix of cofactors} = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}'$$

$$= \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$ **Ans.**

8. If $\sin^2 x + \cos^2 y = 1$, then find $\frac{dy}{dx}$. **2**

Sol. According to question,
$\sin^2 x + \cos^2 x = 1$
Differentiating with respect to x

$$2 \sin x \frac{d}{dx}(\sin x) + 2 \cos y \frac{d}{dx}(\cos y) = 0$$

or $2 \sin x \cos x + 2 \cos y (-\sin y) \frac{dy}{dx} = 0$

$$\therefore \quad \frac{dy}{dx} = \frac{2 \sin x \cos x}{2 \sin y \cos y} = \frac{\sin 2x}{\sin 2y}$$ **Ans.**

9. Differentiate $\log(\cos \cdot e^x)$ with respect to x. **2**

Sol. According to question,
$y = \log(\cos e^x)$
Let $y = \log s, s = \cot t, t = e^x$

$$\frac{dy}{ds} = \frac{1}{s}, \frac{ds}{dt} = -\sin t, \frac{dt}{dx} = e^x$$

$$\frac{dy}{dx} = \frac{dy}{ds} \times \frac{ds}{dt} \times \frac{dt}{dx} = \frac{1}{s} \times (-\sin t) \times e^x$$

Now $s = \cos t$, $t = e^x \therefore s = \cos e^x$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{\cos t}(-\sin e^x) \times e^x = -\frac{e^x \sin e^x}{\cos e^x}$$

$$= -e^x \tan e^x$$

When $e^x \neq (2n+1)\frac{\pi}{2}, n \in Z$ **Ans.**

10. Find $\frac{dy}{dx}$, if $x = 4t, y = \frac{4}{t}$. **2**

Sol. According to question,

$$x = 4t, y = \frac{4}{t}$$

Differentiating w.r.t t

$$\therefore \quad \frac{dx}{dt} = 4 \quad \frac{dy}{dt} = \frac{-4}{t^2}$$

$$\therefore \quad \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$= \frac{-4}{t^2} \times \frac{1}{4} = \frac{-1}{t^2}$$ **Ans.**

11. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in R. **2**

Sol. According to question,

$f(x) = x^3 - 3x^2 + 3x - 100$
$\therefore \quad f(x) = 3x^2 - 6x + 3$
$= 3(x^2 - 2x + 1)$
$= 3(x - 1)^2$
$x \in R, f(x) = +\text{ve}$

Hence, f is an increasing function.

12. Evaluate $\int \sin^3 x \cos^3 x \, dx$. **2**

Sol. Let $I = \int \sin^3 x \cos^3 x \, dx$

$$= \int (1 - \cos^2 x) \cdot \cos^3 x \sin x \, dx$$

Here putting $\cos x = t, -\sin x \, dx = dt$

$$I = -\int (1 - t^2) \cdot t^3 \, dt = -\int (t^3 - t^5) dt$$

$$= \frac{-t^4}{4} + \frac{t^6}{6} + c$$

$$= \frac{1}{6}\cos^6 x - \frac{1}{4}\cos^4 x + c$$ **Ans.**

13. Find the area enclosed by the circle $x^2 + y^2 = a^2$. **2**

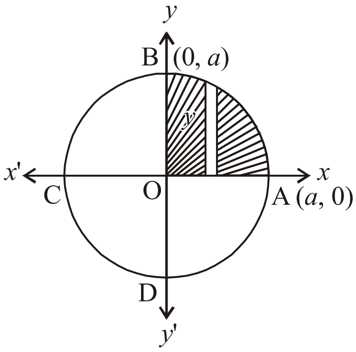
Sol. Let ABCDA be the area of given circle $x^2 + y^2 = a^2$.
By symmetry of circle,
required area = 4 × area of quadrant OABO

$$= 4 \times \int_0^a y \, dx$$

From equation of circle,
$y^2 = a^2 - x^2$

$\Rightarrow \qquad y = \sqrt{a^2 - x^2}$

Therefore the required area $= 4 \times \int_0^a \sqrt{a^2 - x^2}\, dx$



$$= 4\left[\frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2 \sin^{-1}\frac{x}{a}\right]_0^a$$

$$= 4\left[0 + \frac{1}{2}a^2 \sin^{-1} 1 - 0 - 0\right] = 2a^2 \sin^{-1} 1$$

$$= 2a^2 \times \frac{\pi}{2} = \pi a^2 \text{ sq. unit.}$$ **Ans.**

14. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$. **2**

Sol. According to question, the adjacent sides of parallelogram,

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$$

and $$\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$

$$\therefore \quad \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

$$= (-1 + 21)\hat{i} - (1 - 6)\hat{j} + (-7 + 2)\hat{k}$$

$$= 20\hat{i} + 5\hat{j} - 5\hat{k}$$

$\therefore$ Area of parallelogram

$$= |\vec{a} \times \vec{b}| = |20\hat{i} + 5\hat{j} - 5\hat{k}|$$

$$= \sqrt{20^2 + 5^2 + (-5)^2}$$

$$= \sqrt{400 + 25 + 25}$$

$$= \sqrt{400} = 15\sqrt{2} \text{ sq. unit.}$$ **Ans.**

15. A fair die has been tossed. Find P (E/F) and P(F/E) for the events E = {1, 3, 5}, F = {2, 3} and G = {2, 3, 4, 5}. **2**

Sol. A die can show 1, 2, 3, 4, 5 or 6. Therefore there are 6 results in sample space.

$\therefore \quad n(S) = 6$

E = {1, 3, 5}, F {2, 3}, G {2, 3, 4, 5}

$E \cap F = \{3\} \Rightarrow n(E \cap F) = 1$

$$\therefore \quad P(E \cap F) = \frac{1}{6},\ P(E) = \frac{3}{6},\ P(F) = \frac{2}{6}$$

Now $$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{1}{6} \div \frac{2}{6}$$

$$= \frac{1}{2}$$ **Ans.**

$$P\left(\frac{F}{E}\right) = \frac{P(E \cap F)}{P(E)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$$ **Ans.**

Section–C

Long Answer Type Questions :

16. Evaluate $\int \frac{x^2}{\sqrt{x^6 + a^6}}\, dx$. **3**

Or

Evaluate $\int \frac{x}{(x+1)(x+2)}\, dx$. **3**

Sol. Let $I = \int \frac{x^2}{\sqrt{x^6 + a^6}}\, dx$, Now putting $x^3 = t$

$$\therefore\ 3x^2\, dx = dt$$

$$\therefore \quad I = \frac{1}{3}\int \frac{dt}{\sqrt{t^2 + a^6}} = \frac{1}{3}\int \frac{dt}{\sqrt{t^2 + (a^3)^2}}$$

$$= \frac{1}{3} \log \left| t + \sqrt{t^2 + (a^3)^2} \right| + C$$

$$\left[\because \int \frac{1}{\sqrt{x^2 + a^2}}\, dx = \log\left|\left(x + \sqrt{x^2 + a^2}\right)\right|\right]$$

$$\therefore \quad I = \frac{1}{3} \log \left| x^3 + \sqrt{x^6 + a^6} \right| + C$$ **Ans.**

Or

Sol. Let $\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$\therefore \quad x = A(x + 2) + B(x + 1)$

Putting $x = -1$, $\quad -1 = A.\ 1 \qquad \therefore A = -1$

Putting $x = -2$, $\quad -2 = B(-2 + 1) \qquad \therefore B = 2$

$$\therefore \quad \frac{x}{(x+1)(x+2)} = -\frac{1}{x+1} + \frac{2}{(x+2)}$$

$$\therefore \int \frac{x}{(x+1)(x+2)}\, dx = -\int \frac{1}{x+1}\, dx + 2\int \frac{1}{x+2}\, dx$$

$$= -\log|x + 1| + 2\log|x + 2| + C$$

$$= \log \frac{|x + 2|^2}{|x + 1|} + C$$ **Ans.**

17. Find the general solution of the differential equation

$$x\frac{dy}{dx} + 2y = x^2 (x \neq 0). \qquad \textbf{3}$$

Or

Find the general solution of the differential equation $(e^x + e^{-x})\, dy - (e^x - e^{-x})\, dx = 0$. **3**

Sol. Given differential equation is :

$$x\frac{dy}{dx} + 2y = x^2$$

or $$\frac{dy}{dx} + \frac{2}{x}y = \frac{x^2}{x}$$

or $$\frac{dy}{dx} + \frac{2}{x}y = x$$

This is in the form of linear differential equation of kind

$$\frac{dy}{dx} + Py = Q.$$

Here, $P = \frac{2}{x}$ and $Q = x$

$$\text{I.F.} = e^{\int P dx} = e^{\int \frac{2}{x} dx} = e^{2\int \frac{1}{x} dx} = e^{2\log x} = e^{\log x^2} = x^2$$

The solution of the given differential equation :

$$y(\text{I.F.}) = \int ((\text{I.F.}).Q)\, dx + C$$

$$\Rightarrow \quad y \cdot (x^2) = \int (x^2)\,(x)\, dx + C$$

$$\Rightarrow \quad x^2 y = \int x^2 dx + C$$

$$\Rightarrow \quad x^2 y = \frac{x^4}{4} + C$$

or $$y = \frac{x^4}{4} + Cx^{-2}$$

This is the required general solution of given differential equation.

Or

Sol. Given $(e^x + e^{-x})\, dy - (e^x - e^{-x})\, dx = 0$

or $(e^x + e^{-x})\, dy = (e^x - e^{-x})\, dx$

$$\therefore \quad dy = \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$$

or $$\int dy = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$$

$$y = \log (e^x + e^{-x}) + C$$

$$\left[\int \frac{f'(x)}{f(x)}\, dx = \log |f(x)| + C\right]$$

This is the required solution. **Ans.**

18. Find the angle between the pair of lines given by

$\vec{r} = (3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda\,(\hat{i} + 2\hat{j} + 2\hat{k})$ and

$\vec{r} = (5\hat{i} - 2\hat{j}) + \mu\,(3\hat{i} + 2\hat{j} + 6\hat{k})\cdot$ **3**

Or

Show that the line through the point (1, – 1, 2), (3, 4, – 2) is perpendicular to the line through the point (0, 3, 2) and (3, 5, 6). **3**

Sol. Let θ be the angle between vectors $\vec{b_1} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b_2} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, therefore

$$\cos \theta = \left|\frac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}|\,|\vec{b_2}|}\right| = \left|\frac{(\hat{i} + 2\hat{j} + 2\hat{k})\cdot(3\hat{i} + 2\hat{j} + 6\hat{k})}{\sqrt{1+4+4}\sqrt{9+4+36}}\right|$$

$$= \left|\frac{3 + 4 + 12}{3 \times 7}\right| = \frac{19}{21}$$

So, $$\theta = \cos^{-1}\left(\frac{19}{21}\right)$$

Or

Sol. Le the given points be A (1, – 1, 2) and B (3, 4, – 2). So the direction ratios of line paring through the points A (1, – 1, 2) and B (3, 4, – 2) are 3 – 1, 4 + 1, – 2 – 2 or 2, 5, – 4.

Similarly the other given points are C (0, 3, 2) and D (3, 5, 6). So the direction ratios of line passing through the points C (0, 3, 2) and D (3, 5, 6) are 3 – 0, 5 – 3, 6 – 2 or 3, 2, 4.

Now of AB $\perp$ CD, then

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

So, 2 × 3 + 5 × 2 + (– 4) × 4 = 6 + 10 – 16 = 0

Hence the lines parring through given points are perpendicular to each other.

19. A family has two children. What is the probability that both the children are boys given that at least one of them is a boy ? **3**

Or

A die is thrown. If E is the event 'the number appearing is a multiple of 3′ and F be the event 'the number appearing is even' then find whether E and F are independent? **3**

Sol. Let b represent boy and g represent girl. The sample space of experiment is

$$S = \{(b, b), (g, b), (b, g), (g, g)\}$$

Let E and F represent the following events respectively.

E 'Both the children are boy'

F = 'At least one of the children is a boy'.

Then E = {(b, b)} and F = {(b, b), (g, b), (b, g)}

Now E $\cap$ F = {(b, b)}

So, $$P(F) = \frac{3}{4} \text{ and } P(E \cap F) = \frac{1}{4}$$

$$\therefore \quad P(E|F) = \frac{P(E \cap F)}{P(F)}$$

Putting the values

$$P(E|F) = \frac{1/4}{3/4}$$

$$P(E|F) = \frac{1}{3}$$ **Ans.**

Or

Sol. We know that the sample space of given experiment is

$S = \{1, 2, 3, 4, 5, 6\}$

Now $E = \{3, 6\}$, $F = \{2, 4, 6\}$ and $E \cap F = \{6\}$

then $P(E) = \frac{2}{6} = \frac{1}{3}$, $P(F) = \frac{3}{6} = \frac{1}{2}$

and $P(E \cap F) = \frac{1}{6}$

Clearly $P(E \cap F) = P(E) \cdot P(F)$

Hence E and F are independent events.

Section–D

Essay Type Questions :

20. Evaluate $\int \sqrt{(1 - 4x - x^2)}\,dx$. **4**

OR

Evaluate $\int_{-1}^{1} 5x^4 \sqrt{x^5 + 1}\,dx$. **4**

Sol. Let $I = \int \sqrt{1 - 4x - x^2}\,dx$

$$= \int \sqrt{1 - (x^2 + 4x + 4) + 4}\,dx$$

$$= \int \sqrt{5 - (x + 2)^2}\,dx$$

$$\left[\because \int \sqrt{a^2 - x^2}\,dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$$

Putting 5 in place of a^2 and $x + 2$ in place of x

$$\therefore \quad I = \frac{1}{2}(x + 2)\sqrt{5 - (x + 2)^2} + \frac{5}{2}\sin^{-1}\frac{x + 2}{\sqrt{5}} + C$$

$$= \frac{1}{2}(x + 2)\sqrt{1 - 4x - x^2} + \frac{5}{2}\sin^{-1}\frac{x + 2}{\sqrt{5}} + C$$

$$\therefore \quad I = \frac{5}{2}\sin^{-1}\left(\frac{x + 2}{\sqrt{5}}\right) + \frac{x + 2}{2}\sqrt{1 - 4x - x^2} + C$$

Ans.

Or

Sol. Let $I = \int_{-1}^{1} 5x^4 \sqrt{x^5 + 1}\,dx$

Putting $x^5 + 1 = t^2$

$5x^4\,dx = 2t\,dt$

When $x = -1$, then

$t^2 = (-1)^3 + 1 = -1 + 1$

$t = 0$

When $x = 1$ then $t^2 = (1)^5 + 1 = 1 + 1 = 2$

$t = \sqrt{2}$

So, $I = \int_0^{\sqrt{2}} \sqrt{t^2} \cdot 2t\,dt = 2\int_0^{\sqrt{2}} t^2\,dt$

$$= 2\left[\frac{t^3}{3}\right]_0^{\sqrt{2}} = \frac{2}{3}\left[t^3\right]_0^{\sqrt{2}}$$

$$= \frac{2}{3}\left[(\sqrt{2})^3 - 0\right] = \frac{2 \times 2\sqrt{2}}{3}$$

$$= \frac{4\sqrt{2}}{3}$$ **Ans.**

21. Find the shortest distance between the lines l_1 and l_2 whose vector equations are

$$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$$

Or

Find the equation of the line in vector and in Cartesian form that passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction $\hat{i} + 2\hat{j} - \hat{k}$. **4**

Sol. Given lines

$$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k}) \quad ...(1)$$

$$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}) \quad ...(2)$$

Comparing equations (1) and (2) with $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$

$$\vec{a_1} = \hat{i} + \hat{j},\ \vec{b_1} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{a_2} = 2\hat{i} + \hat{j} - \hat{k} \text{ and } \vec{b_2} = 3\hat{i} - 5\hat{j} + 2\hat{k}$$

So, $\vec{a_2} - \vec{a_1} = \hat{i} - \hat{k}$

and $\vec{b_1} \times \vec{b_2} = (2\hat{i} - \hat{j} + \hat{k}) \times (3\hat{i} - 5\hat{j} + 2\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$$= (-2 + 5)\hat{i} - (4 - 3)\hat{j} + (-10 + 3)\hat{k}$$

$$= 3\hat{i} - \hat{j} - \hat{k}$$

Thus $\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{9 + 1 + 49} = \sqrt{59}$

Therefore the shortest distance

$$d = \left|\frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} \times \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|}\right|$$

Putting the values $d = \left|\frac{(3\hat{i} - \hat{j} - 7\hat{k}) \cdot (\hat{i} - \hat{k})}{\sqrt{59}}\right|$

$$= \left|\frac{3-0+7}{\sqrt{59}}\right| = \frac{10}{\sqrt{59}}$$ **Ans.**

Or

Sol. (i) Equation of line passing through point $\vec{a}$ and parallel to vector $\vec{b}$ is :

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

Here $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$

and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$

$\therefore$ The required equation of line in vector form

$$\vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$$ **Ans.**

(ii) Equation of line paring through the point (x_1, y_1, z_1) and having direction ratios a, b, c is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Here the point is $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ or (2 – 1, 4)

$$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$$

$\therefore$ The direction ratio of line are 1, 2, – 1.

$\therefore$ Required equation of line in Cartesian for m is

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$$ **Ans.**

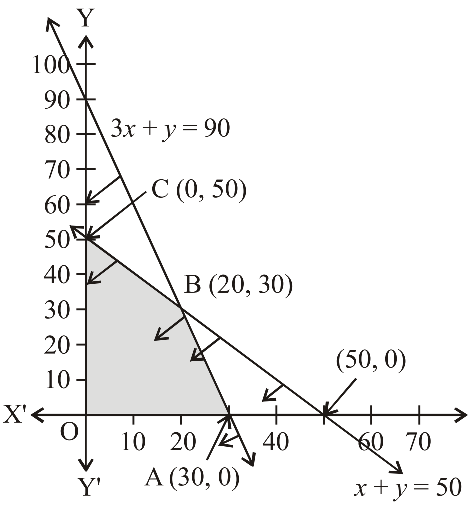
22. Maximize Z = $4x + y$ subject to constraints $x + y \le 50$, $3x + y \le 90$, $x \ge 0$, $y \ge 0$ by using graphical method. **4**

Or

Maximize Z = $3x + 2y$ subject to constraints $x + 2y \le 10$, $3x + y \le 15$, $x \ge 0$, $y \ge 0$ by using graphical method. **4**

Sol. According to question,

$$z = yx + y$$



The constraints are

$x + y \le 50$, $3x + y \le 90$, $x \ge 0$, $y \ge 0$

The shaded ragion in the figure is the coherent region determined by the system of given constraints. We observe that the coherent region OABC is bounded. Therefore we will use the corner point method to find the maximum value of z.

Corner Points	Corresponding value of Z
(0, 0)	Z = $4x + y$ = 4 × 0 + 0 = 0
(30, 0)	Z = $4x + y$ = 4 × 30 + 0 = 120 ← Maximum
(20, 30)	Z = $4x + y$ = 4 × 20 + 30 = 110
(0, 50)	Z = $4x + y$ = 4 × 0 + 50 = 50

The coordinates of corner points O, A, B and C are (0, 0), (30, 0), (20, 30) and (0, 50) respectively.

The maximum value of Z is 120 at point (30, 0). **Ans.**

Or

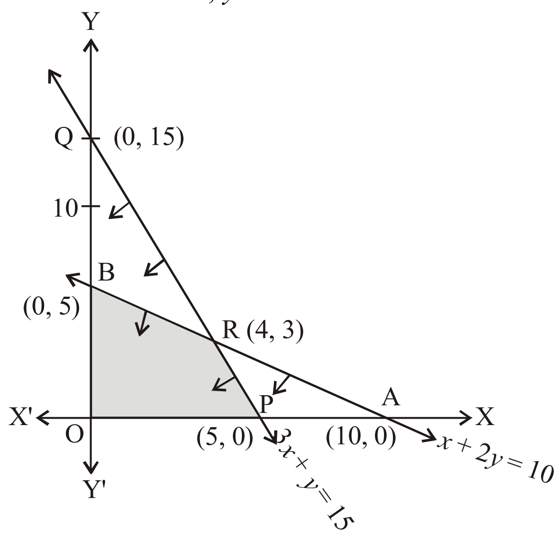
Sol. According to question,

$$Z = 3x + 2y$$

Constrants, $x + 2y \le 10$

$3x + y \le 15$

$x, y \ge 0$



The shaded region in the figure is coherent region determined by the system of given constraints. We observe that the feasible region OPRBO is bounded. Therefore we will use the corner point method to find the value of Z.

Corner Points	Corresponding value of Z
O (0, 0)	Z = 3 × 0 + 2 × 0 = 0
P (5, 0)	Z = 3 × 5 + 2 × 0 = 15
R (4, 3)	Z = 3 × 4 + 2 × 3 = 18 ← Maximum
B (0, 5)	Z = 3 × 0 + 2 × 5 = 10

The coordinates of corner points O, P, R and B are (0, 0), (5, 0), (4, 3) and (0, 5) respectively. The maximum value of Z is 18 at point R (4, 3). **Ans.**

●●

RELATIONS AND FUNCTIONS

CHAPTER 1

Chapter Overview

1.1 Introduction

- In class XI we have studied in detail set, subset, cartesian product, relation, domain and co-domain, range of relation, function, domain, co-domain and range of function. In this chapter we shall recall the above definitions and then study the types of functions in detail.

1.2 Ordered Pair

- Generally on changing the order of the elements of the set, no change occurs in the set. For example $\{1, 2\} = \{2, 1\}$ but if the order of the elements of any set has importance, then such a set is called ordered set. Similarly if in the set $\{a, b\}$ of two elements, a is assigned the first place and b is assigned the second place, then this set is called an ordered pair and it is expressed by the symbol (a, b). Here $(a, b) \neq (b, a)$.

From definition it is clear that

$(a, b) = (c, d) \Leftrightarrow a = c$ and $b = d$

❖ If in any ordered set, the number of element be n, then such a set is called ordered n-tuple and it is expressed by $(a_1, a_2, ..., a_n)$. For example : In two dimension are coordinates (x, y) and three dimensional cordinates (x, y, z), order has its importance.

1.3 Cartesian Product of two sets

❖ The cartesian product of two sets A and B is the set of some ordered pair where first element a is the element of set A and second element b is the element of set B. This product is expressed by the symbol A × B. Hence,

$A \times B = \{(a, b) \mid a \in A, b \in B\}$

❖ By definition it is clear that $A \times B \neq B \times A$ until A and B are equal.

☞ **Example.** If A = {p, q, r} and B = {x, y} then
$A \times B = \{(p, x), (p, y), (q, x), (q, y), (r, x), (r, y)\}$
$B \times A = \{(x, p), (y, p), (x, q), (y, q), (x, r), (y, r)\}$

❖ **Remarks :**

(i) If $A = \phi$ or $B = \phi$, then $A \times B = \phi$ Here ϕ is the null set.

(ii) If $A = \phi$ and $B = \phi$, then $A \times B = \phi$

(iii) If the number of elements in the set A is m and the number of elements in the set B is n, then there will be $m \times n$ elements in A × B. Hence the number of its non-empty subsets will be $2^{mn} - 1$.

(iv) If A and B are non-empty sets and either one or both of them are infinite sets and the number of elements in A × B will be infinite, *i.e.*, A × B also will be an infinite set.

(v) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(vi) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(vii) $A \times (B - C) = (A \times B) - (A \times C)$

(viii) $A \subseteq B \Rightarrow A \times C \subseteq B \times C$

(ix) $A \subseteq B, C \subseteq D \Rightarrow (A \times C) \subseteq (B \times D)$

1.4 Relation

❖ The relation R defined from set A into set B is a subset of A × B, *i.e.*, $R \subseteq A \times B$.

$R = \{(x, y) \mid xRy, x \in A, y \in B\}$

❖ If a and b are related by relation R then this fact can be written as follows :

a R b or $(a,b) \in R$

❖ If a and b are not related by relation R, then we can write this fact as follows :

$a \not R b$ or $(a, b) \notin R$

☞ **Example 1.** If A = {3, 5, 7, 9,}, B = {4, 6, 8, 10} and P(x,y) = x, is smaller than y, then

R = {A, B, P (x, y)}

is a relation from A into B. Under this relation 3R4, 3R6, 3R8, 3R10, 5R6, 5R8, 5R10, 7R8, 7R10, 9R10, but $5 \not R 4$, $7 \not R 4$, $7 \not R 6$, $9 \not R 4$, $9 \not R 6$, $9 \not R 8$,

We can express it also as follows :

$(3,4) \in R$, $(3,6) \in R$, $(3,8) \in R$, $(3,10) \in R$, $(5,6) \in R$, $(5,8) \in R$, $(5,10) \in R$, $(7,8) \in R$, $(7,10) \in R$, $(9,10) \in R$, but $(5,4) \notin R$, $(7,4) \notin R$, $(7,6) \notin R$, $(9,4) \notin R$, $(9,6) \notin R$, $(9,8) \notin R$ etc. Then R = {(3, 4), (3, 6), (3, 8), (3, 10), (5, 6), (5, 8), (5, 10), (7, 8), (7, 10), (9, 10)}

☞ **Example 2.** If A = {2, 3, 4}, B = {3, 6, 8} and P(x,y) = x is divisor of y, then

R = {A, B, P(x,y)}

is a relation from A into B. Under this relation 2R6, 2R8, 3R3, 3R6, 4R8 but $2 \not R 3$, $3 \not R 8$, $4 \not R 3$, $4 \not R 6$, *i.e.* $(2, 6) \in R$, $(2, 8) \in R$, $(3, 3) \in R$, $(3, 6) \in R$, $(4, 8) \in R$, but $(2, 3) \notin R$, $(3, 8) \notin R$, $(4, 3) \notin R$, $(4, 6) \notin R$ etc.

☞ **Example 3.** If A = {1, 2, 3, 5, 7}, B = {1, 4, 6, 9} and P(x,y) : x is double of y, then R = {A, B, P(x, y)} is a relation from A into B, under which 2R4, 3R6 but $1 \not R 4$, $3 \not R 9$, etc. We can express it also as follows :

$(2, 4) \in R$, $(3, 6) \in R$, but $(1, 4) \notin R$, $(3, 9) \notin R$ etc.

❖ **Remarks :** From above examples it is obvious that

(i) It is not necessary that each element of A is related with some or the other element of B, *i.e.*, there may be such elements in a which are not related with any element of B.

(ii) An element of A way be related with one or more elements of B.

(iii) One or more elements of A may be related with one element of B.

(iv) No element of A may be related with any element of B.

(v) All the elements of A may be related with all the elements of B.

❖ **Note :** If the number of elements in A and B be m and n respectively, then the number of elements in A × B will be $m \times n$. The number of its non-empty subsets will be $2^{mn} - 1$, *i.e.*, the number of non-empty relation defined from A into B will be $2^{mn} - 1$.

1.5 Domain and Range of a Relation

❖ If R is any relation defined from set A into set B, then the set of first elements of the ordered pairs of R is called domain of relation R and the set of second elements of the ordered pairs of R is called the range of R. Hence,

Domain of R $= \{a|(a,b)\in R\}$

Range of R $= \{b|(a,b)\in R\}$

From above it is clear that the domain of R will be the subset of A and the range of R will be the subset of B.

☞ **Example 1.** If A = {2, 4, 6, 8}, B={3, 5, 9} and a relation R from A into B is defined such that $x R y \Leftrightarrow$ x is greater than y, then,

R = {(4, 3), (6, 3), (6, 5), (8, 3), (8, 5)}

In the above relation,

Domain of R = {4, 6, 8}

Range of R = {3, 5}

☞ **Example 2.** If A = {1, 2, 3, 4, 5} and B = {2, 4, 6, 8, 10}

and let R = {(a, b)| a∈A, b∈B, a is divisor of b} is a relation from A into B, then

R = {(1, 2), (1, 4), (1, 6), (1, 8), (1, 10), (2, 2), (2, 4), (2, 6), (2, 8), (2, 10), (3, 6), (4, 4), (4, 8), (5, 10)}

Hence domain of R = {1, 2, 3, 4, 5} = A

Range of R = {2, 4, 6, 8, 10} = B

☞ **Example 3.** A relation R is defined in Z by

R = $\{(x,y)|\ x, y \in Z,\ x^2 + y^2 \le 4\}$

domain of R = {–2, –1, 0, 1, 2}

and range of R = {–2, –1, 0, 1, 2}

☞ **Example 4.** If R = {(x, y) : $x + 2y = 8$} is a relation on N, then write the range of R.

Sol. Here $x R y \Leftrightarrow x + 2y = 8$

$$\Leftrightarrow y = \frac{8-x}{2},\ x\in N,\ y\in N$$

when $x = 2,\ y = \frac{8-2}{2} = 3\in N$

$x = 4,\ y = \frac{8-4}{2} = 2\in N$

$x = 6,\ y = \frac{8-6}{2} = 1\in N$

$x = 8,\ y = \frac{8-8}{2} = 0\notin N$

Hence range of R = {1, 2, 3} **Ans.**

1.6 Inverse Relation

❖ Let R be a relation defined from set A into set B. Then the inverse relation R^{-1} of R, from set B into set A is defined as follows :

$$R^{-1} = \{(b, a) \in B \times A : (a, b)\in R\}$$

i.e. $(a, b)\in R \Leftrightarrow (b, a)\in R^{-1}$

or $aRb \Leftrightarrow bR^{-1}a$

By definition it is obvious that

domain of R^{-1} = range of R

and range of R^{-1} = domain of R

☞ **Example 1.** If A = {1, 2, 3}, and B = {0, 4} and relation R from set A into set B is defined such that

R = {(1, 0), (2, 0), (3, 0)}

Then inverse relation of R will be :

R^{-1} = {(0, 1), (0, 2), (0, 3)}

From above it is obvious that :

Domain of R^{-1} = {0} = Range of R

Range of R^{-1} = {1, 2, 3} = Domain of R

☞ **Example 2.** If relation R in N is defined by "x is less than y" then R = $\{(x,y)|x, y\in N, x < y\}$ then, its inverse relation R^{-1} "x is greater than y" is defined by $R^{-1} = \{(x,y)|x, y\in N, x > y\}$.

1.7 Types of Relations

❖ Relation are of the following types :

(i) Reflexive Relation

(ii) Symmetric Relation

(iii) Transitive Relation

(iv) Trivial Relation

1.8 Reflexive Relation

❖ If a relation R is defined in any set A such that under it each element of A is related with itself, then the relation R is called reflexive relation. Hence R is a reflexive relation if and only if

$$aRa \ \forall \ a \in A$$

i.e., R is a reflexive relation

$\Leftrightarrow$ $(a, a) \in R, \ \forall \ a \in A$

❖ From the above definition it is obvious that the relation R defined in A will not be a reflexive relation if there exists at least one element a in A which is not related with itself, *i.e.*, $(a, a) \notin R$

❖ By the definition of reflexive relation R and identity relation I_A defined in any set it is clear that I_A is a subset of R, *i.e.*, it is clear that I_A is a subset of R, *i.e.*, $I_A \subseteq R$

Hence identity relation I_A of any set A is essentially a reflexive relation in A, but it is not necessary that each reflexive relation defined in A is an identity relation.

❖ **Remark :** For a reflexive relation $(a, a) \in R$ but it does not mean that the element a is not related with any other element other than a, *i.e.*, a along with being related with itself, may be related with other elements of A also, where as in the identity relation, a is related with a and only a. Hence it is evident that every identity relation is a reflexive relation but every reflexive relation is not necessarily an identity relation.

☞ **Example 1.** If N is the set of natural nubmers and a relative R is defined in N such that $xRy \Leftrightarrow x$, is divisor of y, $\forall x, y \in N$ then R will be a reflexive relation because every natural number is divisor of itself.

☞ **Example 2.** In the set A of straight lines lying in any plane, if a relation R be defined such that $xRy \Leftrightarrow x, y$. Then R will be a reflexive relation because every line is parallel to itself.

☞ **Example 3.** If in the set B of triangles, a relation R is defined such that

$xRy \Leftrightarrow x$, is congruent to y, then R will be a reflexive relation because each triangle is congruent to itself.

☞ **Example 4.** In the set of sets S if a relation R is defined as follows :

ARB $\Leftrightarrow$ A, is a subset of B, then R will be a reflexive relation because each set is a subset of itself.

☞ **Example 5.** In the set N of natural numbers if a relation R be defined such that $xRy \Leftrightarrow x \geq y$, then R is a reflexive relation because $x \in N \Rightarrow x = x$ but if R is defined such that $xRy \Leftrightarrow x > y$, then this relation will not be reflexive because for any elements of N, $x > x$ is not true.

☞ **Example 6.** Let A = {a, b, c, d} and

R = {(a, a), (a, d), (b, a), (b, b), (c, d), (c, c), (d, d)}

is any relation defined in A, then R is a reflexive relation because (a, a)∈R, (b, b)∈R, (c, c)∈R and (d, d)∈R but if any relation R_1 is defined in A such that R_1 = {(a, a), (a, d), (b, c), (b, d), (c, c), (c, d), (d, b)} then R_1 is not reflexive because b∈A but (b, b)$\notin R_1$. Similarly, d∈A but (d, d) $\notin R_1$

1.9 Symmetric Relation

❖ If a relation R is defined in any set A such that when a is related with b, then b is related with a by the same relation, then the relation R is called symmetric relation. Hence R will be a symmetric relation if and only if $aRb \Rightarrow bRa$, $\forall a, b \in A$, *i.e.*, $(a, b) \in R \Rightarrow (b, a) \in R$, $\forall a, b \in A$

From above it is clear that a relation R defined in the set A will not be symmetric if there exist two elements a and b in A such that

$$a \ R \ b \text{ but } b \not{R} a$$

❖ **Note :** Inverse relation of a symmetric relation R is