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Index

Mathematics—Class 10

Chapter–1

Real Numbers

Important Points

1. **Fundamental Theorem of Arithmetic—**Every composite number can be expressed (factorised) as a product of prime numbers and this factorisation is unique apart from the order in which prime factors occur.
2. We will use the fundamental theorem of arithmetic in two applications—
   (i) To prove the irrationality of some numbers like $\sqrt{2}, \sqrt{3}, \sqrt{5}$ etc.
   (ii) To find the prime factorisation of a given numbers.
3. **Highest Common Factor (HCF)—**The greatest common factor of the given numbers is called the Highest Common Factor (HCF) of those numbers.
   $\therefore$ HCF = Product of the smallest power of each common prime factor.
4. **Least Common Multiple (LCM)—**The smallest common multiple of the given numbers is called the Least Common Multiple (LCM).
   $\therefore$ LCM = Product of the greatest power of each prime factorisation associated with the numbers.
5. **Relation between LCM and HCF of Two Numbers—**The product of LCM and HCF of two numbers is the product of two numbers. *i.e.*
   HCF $(a, b) \times$ LCM $(a, b) = a \times b$
6. (i) HCF $(a, b) = \dfrac{a \times b}{\text{LCM}(a,b)}$

   (ii) LCM $(a, b) = \dfrac{a \times b}{\text{HCF}(a,b)}$

   (iii) $a = \dfrac{\text{HCF}(a,b) \times \text{LCM}(a,b)}{b}$

   (iv) $b = \dfrac{\text{HCF}(a,b) \times \text{LCM}(a,b)}{a}$
7. Consider any rational number that when the decimal expansion of p/q is terminating and when it is non terminating and recurring, we find out by looking at the prime factors of every q of p/q.
8. A number '*s*' is called an irrational number if it cannot be written in the form of p/q, where p and q are integers and $q \neq 0$. For example : $\sqrt{2}, \sqrt{3}, \sqrt{15}\pi, -\dfrac{\sqrt{2}}{\sqrt{3}}$, 0.10110111011110........ etc.
9. If p is a prime number and if p divides a^2, then will also divide a, where a is a positive integer.

Irrational numbers : The numbers whose decimal expansions are non-terminating non-recurring are called irrational numbers.

For example : $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \pi$ etc

10. The sum or difference of a rational and an irrational number is irrational.

11. The product and quotient of a non-zero rational and irrational number is irrational.

Textual Questions

EXERCISE 1.1

1. Express each number as a product of its prime factors :

(i) 140 (ii) 156 (iii) 3825
(iv) 5005 (v) 7429

Solution—

(i) Prime factorisation of 140 = 2×70
$= 2 \times 2 \times 35$
$= 2 \times 2 \times 5 \times 7$
$= 2^2 \times 5 \times 7$

(ii) Prime factorisation of 156 = 2×78
$= 2 \times 2 \times 39$
$= 2 \times 2 \times 3 \times 13$
$= 2^2 \times 3 \times 13$

(iii) Prime factorisation of 3825
$= 3 \times 1275$
$= 3 \times 3 \times 425$
$= 3 \times 3 \times 5 \times 85$
$= 3 \times 3 \times 5 \times 5 \times 17$
$= 3^2 \times 5^2 \times 17$

(iv) Prime factorisation of 5005
$= 5 \times 1001$
$= 5 \times 7 \times 143$
$= 5 \times 7 \times 11 \times 13$

(v) Prime factorisation of 7429
$= 17 \times 437$
$= 17 \times 19 \times 23$

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91 (ii) 510 and 92
(iii) 336 and 54

Solution : (i) 26 and 91

Prime factorisation of 26 = 2×13
Prime factorisation of 91 = 7×13

$\therefore$ LCM of 26 and 91 = $2 \times 7 \times 13 = 182$
and HCF of 26 and 91 = 13

Verification—HCF (26, 91) × LCM (26, 91)
$= 13 \times 182$
$= 13 \times 2 \times 91$
$= 26 \times 91$
= Product of given numbers

(ii) 510 and 92

Prime factorisation of 510 = 2×255
$= 2 \times 3 \times 85$
$= 2 \times 3 \times 5 \times 17$

and Prime factorisation of 92 = 2×46
$= 2 \times 2 \times 23$
$= 2^2 \times 23$

LCM (510, 92) = $2^2 \times 3 \times 5 \times 17 \times 23$
= 23460

and HCF (510, 92) = 2

Verification—

HCF (510, 92) × LCM (510, 92)
$= 2 \times 23460$
$= 2 \times 2^2 \times 3 \times 5 \times 17 \times 23$
$= 2 \times 3 \times 5 \times 17 \times 2^2 \times 23$
$= 510 \times 92$
= Product of given numnbers

(iii) 336 and 54

Prime factorisation of 336 = 2×168
$= 2 \times 2 \times 84$
$= 2 \times 2 \times 2 \times 42$
$= 2 \times 2 \times 2 \times 2 \times 21$
$= 2 \times 2 \times 2 \times 2 \times 3 \times 7$
$= 2^4 \times 3 \times 7$

Prime factorisation of 54 = 2×27
$= 2 \times 3 \times 9$
$= 2 \times 3 \times 3 \times 3$
$= 2 \times 3^3$

$\therefore$ HCF (336, 54) = $2 \times 3 = 6$
LCM = $2^4 \times 3^3 \times 7$
= 3024

Verification—

HCF (336, 54) × LCM (336, 54)

$= 6 \times 3024$
$= 2 \times 3 \times 2^4 \times 3^3 \times 7$
$= 2^4 \times 3 \times 7 \times 2 \times 3^3$
$= 336 \times 54$
= Product of given numbers

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29
(iii) 8, 9 and 25

Solution : (i) 12, 15 and 21

Prime factorisation of $12 = 2 \times 2 \times 3$
$= 2^2 \times 3$

Prime factorisation of $15 = 3 \times 5$

Prime factorisation of $21 = 3 \times 7$

$\therefore$ LCM (12, 15 and 21) $= 2^2 \times 3 \times 5 \times 7$
$= 420$ **Ans.**

and HCF (12, 15 and 21) = 3 **Ans.**

(ii) 17, 23 and 29

Prime factorisation of $17 = 1 \times 17$

Prime factorisation of $23 = 1 \times 23$

Prime factorisation of $29 = 1 \times 29$

$\therefore$ LCM (17, 23 and 29) $= 17 \times 23 \times 29$
$= 11339$ **Ans.**

and HCF (17, 23 and 29) = 1 **Ans.**

(iii) 8, 9 and 25

Prime factorisation of $8 = 2 \times 2 \times 2 = (2)^3$

Prime factorisation of $9 = 3 \times 3 = (3)^2$

Prime factorisation of $25 = 5 \times 5 = (5)^2$

$\therefore$ LCM (8, 9 and 25) $= (2)^3 \times (3)^2 \times (5)^2$
$= 8 \times 9 \times 25$
$= 1800$ **Ans.**

and HCF (8, 9 and 25) = 1 **Ans.**

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Solution : According to the question, the numbers are 306 and 657.

$\therefore$ $a = 306$
$b = 657$
and HCF = 9 [Given]

We know that

$$\text{LCM} = \frac{a \times b}{\text{HCF}}$$

$$= \frac{306 \times 657}{9}$$
$= 34 \times 657$
$= 22338$

Hence LCM (306, 657) = 22338 **Ans.**

5. Check whether 6^n can end with the digit 0 for any natural number n.

Solution : Suppose that for any natural number n, $n \in N$, 6^n ends with the digit 0. Hence 6^n will be divisible by 5.

But prime factorisation of $6 = 2 \times 3$

$\therefore$ The prime factorisation of $(6)^n$ will be

$$(6)^n = (2 \times 3)^n$$

i.e., it is clear that there is no place of 5 in the prime factors of 6^n.

$\therefore$ 5 can not be a factor of 6^n

Hence, 6^n can not end with the digit 0 for any natural number n.

6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Solution : According to the question

$7 \times 11 \times 13 + 13 = 13 (7 \times 11 + 1)$

Since 13 is a factor of this number obtained, therefore it is a composite number. Again according to the question

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$
$= 5 (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

This obtained number is also a composite number because it has also a factor 5. Hence both the given numbers are composite numbers.

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Solution : Time taken by Sonia to drive one round of the field = 18 minutes.

Time taken by Ravi to drive one round of the same field = 12 minutes

In order to find after how much time will they meet again at the starting point, we shall have to find out the LCM of 18 and 12. Therefore

prime factorisation of 18 $= 2 \times 9$
$= 2 \times 3 \times 3$
$= 2 \times 3^2$

and prime factorisation of 12 $= 2 \times 6$
$= 2 \times 2 \times 3$
$= 2^2 \times 3$

Taking the product of the greatest power of each prime factor of 18 and 12

$$\text{LCM }(18, 12) = 2^2 \times 3^2 = 4 \times 9 = 36$$

i.e., Sonia and Ravi will meet again at the starting point after 36 minutes. **Ans.**

EXERCISE 1.2

1. Prove that $\sqrt{5}$ is irrational.

Solution : Let us suppose that $\sqrt{5}$ is a rational number. So, we can find two integers r and s where $s \neq 0$ and

$$\sqrt{5} = \frac{r}{s}$$

Now again let us suppose that r and s have some common factors other than 1. Then on dividing r and s by that common factor, we can obtain $\sqrt{5} = \frac{a}{b}$. Here a and b are coprime, *i.e.*

$$b\sqrt{5} = a$$

squaring both sides

$$\Rightarrow \quad \left(b\sqrt{5}\right)^2 = a^2$$

$$\Rightarrow \quad b^2\left(\sqrt{5}\right)^2 = a^2$$

$$\Rightarrow \quad 5b^2 = a^2 \quad .....(i)$$

So 5 divides a^2.

According to theorem if a prime number p divides a^2, then p will divide a also, where a is a positive integer.

$\Rightarrow$ 5 divides a also. .....(ii)

so $a = 5c$ where c in any integer

Putting the value of a in (i)

$$5b^2 = (5c)^2$$

$$5b^2 = 25c^2$$

$$b^2 = 5c^2$$

or $5c^2 = b^2$

$\Rightarrow$ 5 divides b^2

According to theorem 1.2, 5 divides b also. .....(iii)

From (ii) and (iii), there is at least one common factor 5 of a and b. But it is contradicts the fact that a and b are coprime or these have no common factor other than 1. Therefore our hypothesis that $\sqrt{5}$ is a rational number is wrong, *i.e.*, $\sqrt{5}$ in an irrational number. **Hence Proved**

2. Prove that $3 + 2\sqrt{5}$ is irrational.

Solution : Suppose that $3 + 2\sqrt{5}$ is a rational number. So we can find coprime numbers a and b where a and b are integers such that $b \neq 0$ and

$$3 + 2\sqrt{5} = \frac{a}{b}$$

$$\text{or} \quad \frac{a}{b} - 3 = 2\sqrt{5}$$

$$\text{or} \quad \sqrt{5} = \frac{a}{2b} - \frac{3}{2}$$

$$\text{or} \quad \sqrt{5} = \frac{a-3b}{2b}$$

Since a and b both are integers,

$\therefore \frac{a-3b}{2b}$ is a rational number

Therefore $\sqrt{5}$ is a rational number. But this fact contradicts the statement that $\sqrt{5}$ is an irrational number. Therefore this hypothesis is wrong. Therefore the given number $3 + 2\sqrt{5}$ is an irrational number.

Hence Proved

3. Prove that the following are irrationals :

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

Solution : (i) $\frac{1}{\sqrt{2}}$

Contrary to the statement given in question let us assume that $\frac{1}{\sqrt{2}}$ is a rational number. So we can find coprime integers *a* and *b* ($b \neq 0$), *i.e.*,

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

or $$\frac{\sqrt{2}}{2} = \frac{a}{b}$$

or $$\sqrt{2} = \frac{2a}{b} \quad .....(i)$$

Since the quotient of two integers is a rational number, therefore $\frac{2a}{b}$ = a rational number.

From (i) $\sqrt{2}$ also is a rational number. But this statement is wrong. *i.e.*, our hypothesis is incorrect. Therefore $\frac{1}{\sqrt{2}}$ is an irrational number. **Hence Proved**

(ii) $7\sqrt{5}$

Suppose that the given number $7\sqrt{5}$ is a rational number. Therefore we can find two coprime integers *a* and *b* ($b \neq 0$) such that

$$7\sqrt{5} = \frac{a}{b}$$

or $$7b\sqrt{5} = a$$

or $$\sqrt{5} = \frac{a}{7b} \quad .....(i)$$

Since in (i) *a*, 7 and *b* all are integers and the quotient of two integers is also a rational number, *i.e.*,

$\frac{a}{7b}$ = a rational number therefore from (i)

$\sqrt{5}$ = a rational number which is a contradictory statement of the statement '$\sqrt{5}$ is one irrational number.' Hence our assumption in incorrect. Therefore $7\sqrt{5}$ is an irrational number. **Hence Proved**

(iii) $6 + \sqrt{2}$

Suppose that $6 + \sqrt{2}$ in a rational number. So we can find coprime numbers *a* and *b* ($b \neq 0$) such that

$$6 + \sqrt{2} = \frac{a}{b}$$

or $$\sqrt{2} = \frac{a}{b} - 6$$

or $$\sqrt{2} = \frac{a - 6b}{b} \quad .... (i)$$

As *a* and *b* are integers, so $\frac{a-6b}{b}$ will be a rational number because the quotient of integers is also a rational number, *i.e.*,

$$\frac{a-6b}{b} = \text{a rational number}$$

$\therefore$ From (i)

$$\sqrt{2} = \text{a rational number}$$

But this is a contradictory statement of the statement that '$\sqrt{2}$ is an irrational number'.

Hence our assumption is incorrect, *i.e.*, $6 + \sqrt{2}$ is an irrational number.

Hence Proved

Other Important Questions

Objective Type Questions—

1. Which of the following is an irrational number?

(A) 2 (B) 2.232425.....

(C) $2.\overline{23}$ (D) 2.23

2. Given that HCF (156, 78) = 78, then LCM (156, 78) is—

(A) 78 (B) 156

(C) 156 × 78 (D) None of these

3. The prime factorisation of 225 can be written in the following form—

(A) $5^2 \times 3^2$ (B) 5×3^2