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Mathematics

GANITA PRAKASH

For the Student of Class 8

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CHAPTER 1

A SQUARE AND A CUBE

Introduction

Queen Ratnamanjuri had a will written that described her fortune of *ratnas* (precious stones) and also included a puzzle. Her son Khoisnam and their 99 relatives were invited to the reading of her will. She wanted to leave all of her *ratnas* to her son, but she knew that if she did so, all their relatives would pester Khoisnam forever. She hoped that she had taught him everything he needed to know about solving puzzles. She left the following note in her will–

"I have created a puzzle. If all 100 of you answer it at the same time, you will share the *ratnas* equally. However, if you are the first one to solve the problem, you will get to keep the entire inheritance to yourself. Good luck."

The minister took Khoisnam and his 99 relatives to a secret room in the mansion containing 100 lockers.

The minister explained–"Each person is assigned a number from 1 to 100.

- Person 1 opens every locker.
- Person 2 toggles every 2nd locker (*i.e.,* closes it if it is open, opens it if it is closed).
- Person 3 toggles every 3rd locker (3rd, 6th, 9th, ... and so on).
- Person 4 toggles every 4th locker (4th, 8th, 12th, ... and so on).

This continues until all 100 get their turn.

In the end, only some lockers remain open. The open lockers reveal the code to the fortune in the safe."

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Q. 1. Before the process begins, Khoisnam realises that he already knows which lockers will be open at the end. How did he figure out the answer?

Hint : Find out how many times each locker is toggled.

Solution : Khoisnam figured out the answer because he realised that only lockers corresponding to perfect square numbers would remain open at the end. The lockers that will be open are 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100.

Q. 2. Does every number have an even number of factors?

Solution : No, every number has not an even number at factors. Perfect squares have an odd number at factors.

1 :	4 :	9 :
1 × 1	1 × 4	1 × 9
The only factor	2 × 2	3 × 3
is 1.	Factors are	Factors are
	1, 2 and 4.	1, 3 and 9.

Q. 3. Can you use this insight to find more numbers with an odd number of factors?

Solution : Numbers that have an odd number of factors are perfect squares. We can find more such numbers by identifying perfect squares, such as 1, 4, 9, 16, 25, 49, 64, 100, and so on.

Q. 4. Write the locker numbers that remain open.

Solution : The locker numbers that remain open :

1, 4, 9, 16, 25, 36, 49, 64, 81, 100

Q. 5. Khoisnam immediately collects word clues from these 10 lockers and reads, "The passcode consists of the first five locker numbers that were touched exactly twice."

Which are these five lockers?

Solution : The lockers that are toggled twice are the prime numbers, since each prime number has 1 and the number itself as factors. So, the code is 2-3-5-7-11.

Square Numbers

A number that can be expressed as the product of a number with itself is called a square number.

We use the following notation for squares.

$$1 \times 1 = 1^2 = 1$$
$$2 \times 2 = 2^2 = 4$$
$$3 \times 3 = 3^2 = 9$$
$$4 \times 4 = 4^2 = 16$$
$$5 \times 5 = 5^2 = 25.$$
$$\vdots$$

In general, for any number n, we write $n \times n = n^2$, which is read as 'n squared'. Can we have a square of sidelength $\frac{3}{5}$ or 2·5 units?

Yes, there area in square units are $\left(\frac{3}{5}\right)^2 = \left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) = \left(\frac{9}{25}\right)$,

and $(2{\cdot}5)^2 = (2{\cdot}5) \times (2{\cdot}5) = 6{\cdot}25$.

Perfect Squares Numbers

The squares of natural numbers are called perfect squares.
Example : 1, 4, 9, 16, 25, ... are all perfect squares.

Patterns and Properties of Perfect Squares

Square of the first 30 natural numbers.

$1^2 = 1$	$11^2 = 121$	$21^2 = 441$
$2^2 = 4$	$12^2 = 144$	$22^2 = 484$
$3^2 = 9$	$13^2 = 169$	$23^2 = 529$
$4^2 = 16$	$14^2 = 196$	$24^2 = 576$
$5^2 = 25$	$15^2 = 225$	$25^2 = 625$
$6^2 = 36$	$16^2 = 256$	$26^2 = 676$
$7^2 = 49$	$17^2 = 289$	$27^2 = 729$
$8^2 = 64$	$18^2 = 324$	$28^2 = 784$
$9^2 = 81$	$19^2 = 361$	$29^2 = 841$
$10^2 = 100$	$20^2 = 400$	$30^2 = 900$

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Q. 1. What patterns do you notice ? Share your observations and make conjectures.
Solution : (i) The squares of numbers ending in 1 or 9 have a units digit of 1.
Example : $1^2 = 1$; $9^2 = 81$; $11^2 = 121$; $19^2 = 361$; $21^2 = 441$; $29^2 = 841$
(ii) The squares of numbers ending in 2 or 8 have a units digit of 4.
Example : $2^2 = 4$; $8^2 = 64$; $12^2 = 144$; $18^2 = 324$; $22^2 = 484$; $28^2 = 784$
(iii) The squares of numbers ending in 3 or 7 have a units digit of 9.
Example : $3^2 = 9$; $7^2 = 49$; $13^2 = 169$; $17^2 = 289$; $23^2 = 529$; $27^2 = 729$
(iv) The squares of numbers ending in 4 or 6 have a units digit of 6.
Example : $4^2 = 16$; $6^2 = 36$; $14^2 = 196$; $16^2 = 256$; $24^2 = 576$; $26^2 = 676$
(v) The squares of numbers ending in 5 have a units digit of 5.
Example : $5^2 = 25$; $15^2 = 225$; $25^2 = 625$

Q. 2. What are the digit in the units places of these numbers?
Solution : All these numbers end with 0, 1, 4, 5, 6 or 9. None of them end with 2, 3, 7, or 8.

Q. 3. If a number ends in 0, 1, 4, 5, 6 or 9, is it always a square?
Solution : The numbers 16 and 36 are both squares with 6 in the units place. However, 26, whose units digit is also 6, is not a square. Therefore, we cannot determine if a number is a square just by looking at the digit in the units place. But, the units digit can tell us when a number is not a square. If a number ends with 2, 3, 7, or 8, then we can definitely say that it is not a square.

Q. 4. Write 5 numbers such that you can determine by looking at their units digit that they are not squares.

Solution : 12 (ends in 2)
23 (ends in 3)
37 (ends in 7)
48 (ends in 8)
52 (ends in 2)

Q. 5. Let us consider square numbers ending in 6 : $16 = 4^2$, $36 = 6^2$, $196 = 14^2$, $256 = 16^2$, $576 = 24^2$, and $676 = 26^2$.

Solution : The numbers whose squares end in 6 always have a unit digit of either 4 or 6. This is because $4^2 = 16$ and $6^2 = 36$ both ending in 6.

Q. 6. Which of the following numbers have the digit 6 in the units place?
(i) 38^2, (ii) 34^2, (iii) 46^2, (iv) 56^2, (v) 74^2, (vi) 82^2.

Solution : (i) $38^2 = 1444$; The unit digit of $38^2 = 4$
(ii) $34^2 = 1156$; The unit digit of $34^2 = 6$
(iii) $46^2 = 2116$; The unit digit of $46^2 = 6$
(iv) $56^2 = 3136$; The unit digit of $56^2 = 6$
(v) $74^2 = 5476$; The unit digit of $74^2 = 6$
(vi) $82^2 = 6724$; The unit digit of $82^2 = 4$

Therefore, the numbers that have the digit 6 in the units place when squared are (ii) 34^2, (iii) 46^2, (iv) 56^2 and (v) 74^2.

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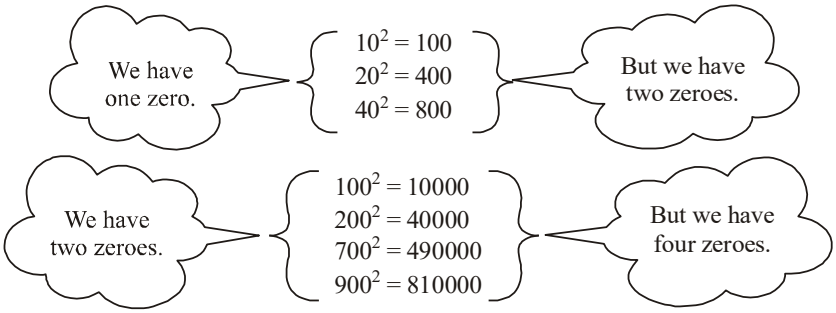
Q. 1. Find more such patterns by observing the numbers and their squares from the table you filled earlier.

Consider the following numbers and their squares.



Solution : Numbers with one zero :
When a number ending in one zero is squared, its square ends in two zeros.
Example : $10^2 = 100$; $20^2 = 400$
Numbers with two zeros :
When a number ending in two zeros is squared, its square ends in four zeros.
Example : $100^2 = 10000$; $200^2 = 40000$